# Neuronal Bifurcation Analysis and Construction of an Oscillatory Associative Memory

Catherine Zeng

## Part I: Bifurcation Analysis

Finding a model of the neuron that is both computationally simple yet biologically plausible such that the neuron is capable of generating observed behaviors of biological neurons is important for understanding the complex neuronal dynamics that emerge from simple rules. Because neurons exist near critical points at which bifurcations happen, bifurcation analysis is useful because it allows us to understand the excitation properties of neurons; although there are millions of electrophysiological mechanisms for excitability and spiking, there are only four different types of bifurcations that the system goes through. By comparing large-scale numerical simulations of brain models with experimental recordings of patterns in the brain, we can understand neural models without knowing the exact details of their complex dynamics.

The first part of this project is to simulate neuronal dynamics using a quadratic integrate-and-fire model of the neuron that contains four dimensionless parameters (Izhikevich 2014). We achieve this by tuning the parameters in response to bifurcation analysis of phase portraits. In this model, $v$ is the membrane potential, $u$ is the recovery current, $C$ is the membrane capacitance, $v_r$ is the resting membrane potential, and $v_t$ is the instantaneous threshold potential.

$$C\frac{\partial v}{\partial t} = k(v - v_r)(v - v_t) - u + I \quad \text{if } v \geq v_{peak} \tag{1}$$

$$\frac{\partial u}{\partial t} = a[b(v - v_r) - u] \quad v \leftarrow c, u \leftarrow u + d \tag{2}$$

The purpose here is to gain some intuition about how to reproduce the spiking and bursting behaviors of known types of neurons by relating bifurcations in phase portraits to experimental recordings (such as the six classes of firing patterns recorded in the mammalian neocortex (Gibson et al. 1999)). This part of the project will build intuition that assists with analysis in the second part of the project.

## Part II: Oscillatory Associative Memory

The Hopfield network, based on the Ising model of ferromagnetism, is currently the most well-known model of associative memory. However, it is important to find alternative models capable of pattern recognition for studying memory and learning because the Hopfield network scales poorly in large simulations; it has an asymptotic time complexity of $O(n^2)$ (every node is connected to every other node). An alternative model of associative memory uses limit cycle attractors to store information, which are generated by Andronov-Hopf bifurcations, rather than point attractors. This relatively unexplored model encodes patterns as constants of coupling in oscillators and is supported by experimental recordings that show the synchronization of neuronal firings plays an important role in information processing for the olfactory bulb, hippocampus, and thalamo-cortical system (Hoppensteadt and Izhikevich 2000).

The second part of the project is to construct an oscillatory associative memory model and evaluate it relative to the Hopfield network for advantages such as network capacity, operating time, and interference levels. Two methods for encoding patterns into oscillators that we explore are frequency shift keying (FSK) and phase shift keying (PSK), which encode patterns into oscillators through frequency shifts and phase differences respectively (Nikonov et al. 2013); for simulating the oscillators, we use the Kuramoto model, which views coupled oscillators in terms of their phase (Acebron et al. 2005). Finally, we evaluate our oscillatory associative memory model through hardware constraints and explore physical implementations such as phase-locked loops (PLL) and nanoscale devices such as nanotransistors and spin torque oscillators.

## References

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