6.004 Tutorial Problems
L06 – Boolean Algebra and Logic Synthesis

Note: A small subset of essential problems are marked with a red star (★). We especially encourage you to try these out before recitation.

A	B	C	F	G
0	0	0	1	1
0	0	1	1	1
0	1	0	0	1
0	1	1	1	0
1	0	0	1	1
1	0	1	0	0
1	1	0	0	1
1	1	1	1	0

Problem 1. ★

Consider the truth table on the right, which defines two functions F and G of three input variables (A, B, and C).

For each function, write it in **normal form**, then find a **minimal sum of products** (minimal SOP) expression.

Normal form for F(A,B,C) = ____________________

Minimal sum of products for F(A,B,C) = ____________________

Normal form for G(A,B,C) = ____________________

Minimal sum of products for G(A,B,C) = ____________________

Problem 2. ★

Consider the 3-input Boolean function $G(A,B,C) = \overline{A} \cdot \overline{C} + A \cdot \overline{B} + \overline{B} \cdot \overline{C}$

1. How many 1's are there in the output column of G's 8-row truth table?

2. Give a minimal sum-of-products expression for G.

3. There's good news and bad news: the bad news is that the stockroom only has G gates. The good news is that it has as many as you need. Using only combinational circuits built from G gates, one can implement (choose the best response):

   (A) Any Boolean function (G is functionally complete)
   (B) Only functions with 3 inputs or less
   (C) Only functions with the same truth table as G

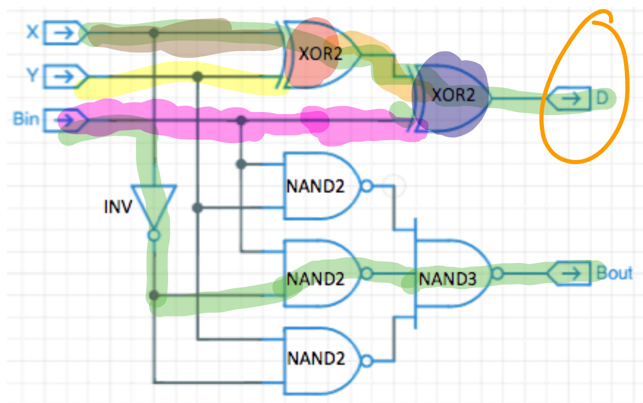
4. Can a sum-of-products expression involving 3 input variables with greater than 4 product terms *always* be simplified to a sum-of-products expression using fewer product terms?

$\bar{A}\bar{B}C + \bar{A}B\bar{C} + A\bar{B}\bar{C} + ABC$

0 0 1   0 1 0

Problem 3. ★

Consider the logic diagram shown below, which includes XOR2, NAND2, NAND3, and INV (inverter) gates.



Gate	t_{PD}
INV	1.0ns
NAND2	1.5ns
NAND3	1.8ns
XOR2	2.5ns

1. Using the t_{PD} information for the gate components shown in the table above, compute the t_{PD} for the circuit.

X → XOR2 → XOR2 → D

2.5 + 2.5 = 5 ns

Bin → inv → NAND2 → NAND3 → Bout

1 + 1.5 + 1.8 = 4.3ns

t_{PD} = 5 ns

2. Find minimal sum-of-products expressions for both outputs, **D** and **Bout**.

NOTE: The gates implement the following functions:

- $NAND2\,(a, b) = \overline{a \cdot b}$
- $NAND3\,(a, b, c) = \overline{a \cdot b \cdot c}$
- $XOR2\,(a, b) = a \cdot \bar{b} + \bar{a} \cdot b$

XOR2 (XOR2(X, Y) , Bin)

$x \cdot \bar{y} + \bar{x} y$

$(x \cdot \bar{y} + \bar{x} y)\overline{Bin} + \overline{(x \cdot \bar{y} + \bar{x} y)}\ Bin$

$x\bar{y}\,\overline{Bin} + \bar{x} y\,\overline{Bin} + xy\,Bin + \bar{x}\bar{y}\,Bin$

$\overline{x \cdot \bar{y}} \cdot \overline{\bar{x} y}$

$(\bar{x} + y) \cdot (x + \bar{y})$

$\bar{x} \cdot x + \bar{x}\bar{y} + xy + y\bar{y}$

$= xy + \bar{x} \cdot \bar{y}$

Minimal sum of products for D(X,Y,Bin) = ____________________

Minimal sum of products for Bout(X,Y,Bin) = ____________________

Problem 4.

Simplify the following Boolean expressions by finding a *minimal sum-of-products expression* for each one:

1. $\overline{ac + b + c}$

2. $(a + b)\,c + \overline{c}a + b(\overline{a} + c)$

3. $a\overline{(b + c)}(b + a(b + c))$

4. $a(b + c\,(d + ef))$

Problem 5.

There are some Boolean expressions for which no assignment of values to variables can produce True (e.g., $a\bar{a}$). Those Boolean expressions are said to be *non-satisfiable*. Are the following Boolean expressions satisfiable? If the expression is satisfiable, give an assignment to variables that makes the expression evaluate to True. If the expression is non-satisfiable, prove it.

1. $(a + b)\,c + \bar{c}a + b(\bar{a} + c)$

2. $(x + y)(x + \bar{y})(z + \bar{y})(y + \bar{x})$

3. $(x + y + z)\,(x + y + \bar{z})\,(x + \bar{y} + z)\,(\bar{x} + y + z)\,\bullet$
   $(x + \bar{y} + \bar{z})(\bar{x} + y + \bar{z})(\bar{x} + \bar{y} + z)(\bar{x} + \bar{y} + \bar{z})$

4. $\overline{xyz + xy\bar{z} + x\bar{y}z + x\overline{yz} + \bar{x}yz + \bar{x}y\bar{z} + \overline{xyz}}$

Problem 6.

(A) Simplify the following Boolean expressions by finding a minimal sum-of-products expression for each one. (*Note:* These expressions can be reduced into a minimal SOP by repeatedly applying the Boolean algebra properties we saw in lecture.)

1. $\overline{(a + b\cdot\overline{c})}\cdot d + c$

2. $a \bullet \overline{(b + c)}(c + a)$

(B) There are Boolean expressions for which no assignment of values to variables can produce True (e.g., $a\cdot\overline{a}$). These Boolean expressions are said to be *non-satisfiable*.

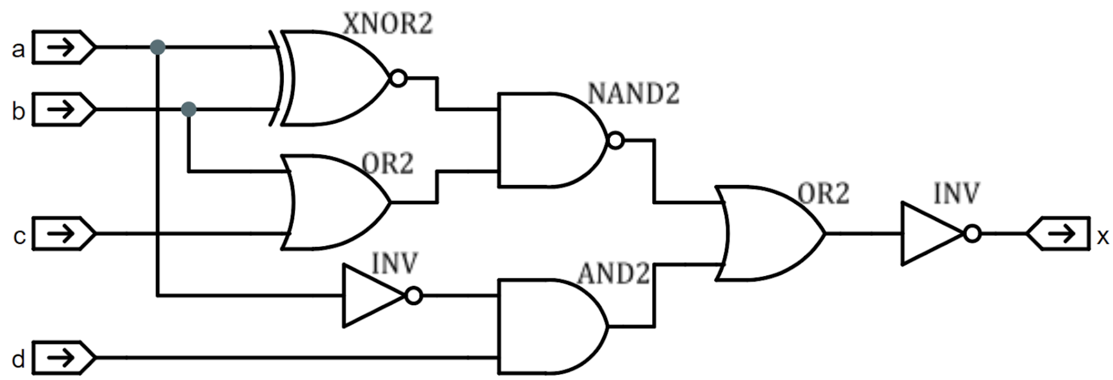
Are the following Boolean expressions satisfiable? If the expression is satisfiable, give an assignment to variables that makes the expression evaluate to True. If the expression is non-satisfiable, explain why.

1. $(\overline{x} + y\overline{z}) \bullet (\overline{y}x + z) \bullet (\overline{z}y + x)$

2. $(\overline{x} + y\overline{z}) \bullet (\overline{y}x + z) \bullet (\overline{z}y + x) + (\overline{x} + yz) \bullet (\overline{y}x + z) \bullet (\overline{z}y + x)$

Problem 7. Boolean Algebra and Combinational Logic (19 points, Spring 2020 Quiz 1) ★

(A) (3 points) Consider the logic diagram below, which includes XNOR2, OR2, NAND2, AND2, and INV. Using the t_{PD} information for the gate components shown in the table below, compute the t_{PD} for the circuit.

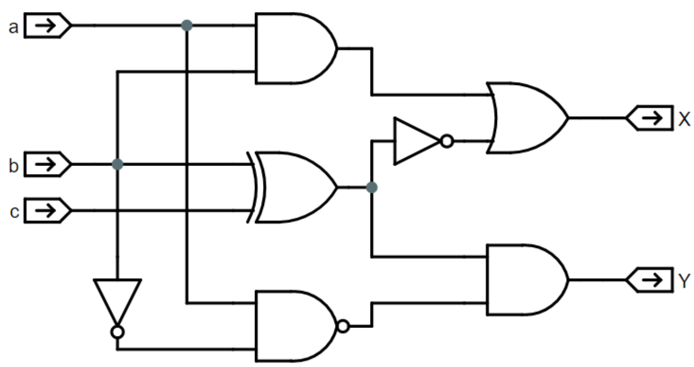


Gate	t_{PD}
XNOR2	7.0ns
OR2	5.5ns
NAND2	3.0ns
AND2	5.0ns
INV	2.0ns

t_{PD} (ns) = ________________________________

(B) (6 points) Given the circuit shown below, construct the truth table for outputs **X** and **Y.**

a	**b**	**c**	**X**	**Y**
0	0	0		
0	0	1		
0	1	0		
0	1	1		
1	0	0		
1	0	1		
1	1	0		
1	1	1		



(C) (4 points) Find a minimal sum-of-products expression for output **X** of the circuit described by the truth table shown below.

a	**b**	**c**	**d**	**X**
0	0	0	0	0
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	0
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

Minimal sum of products for X = ____________________________

(D) (6 points) For each of the following expressions determine if it is satisfiable. If satisfiable, provide a minimal sum-of-products. Otherwise, show why it is not satisfiable.

1. $\overline{\bar{c}\,(a+b)\,(a+d)}(ab\bar{c})$

2. $(x+y)\,(x\bar{y}z+y\bar{z}+\bar{y})$