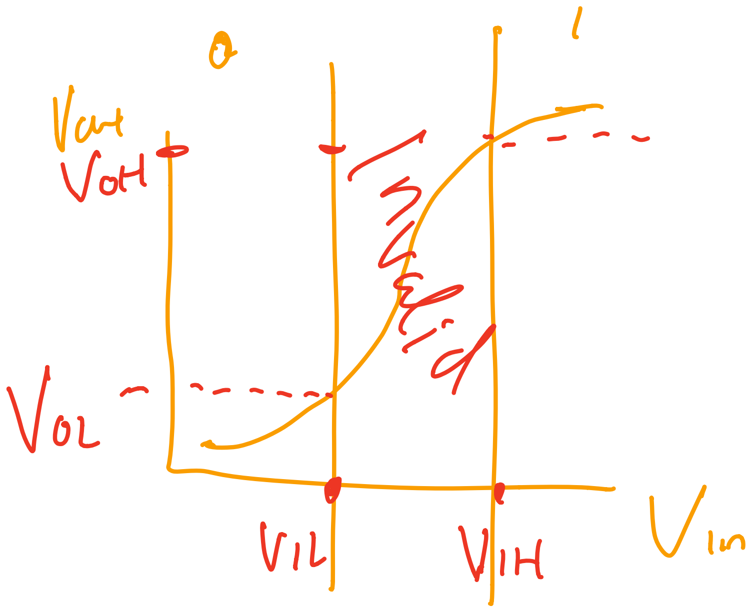
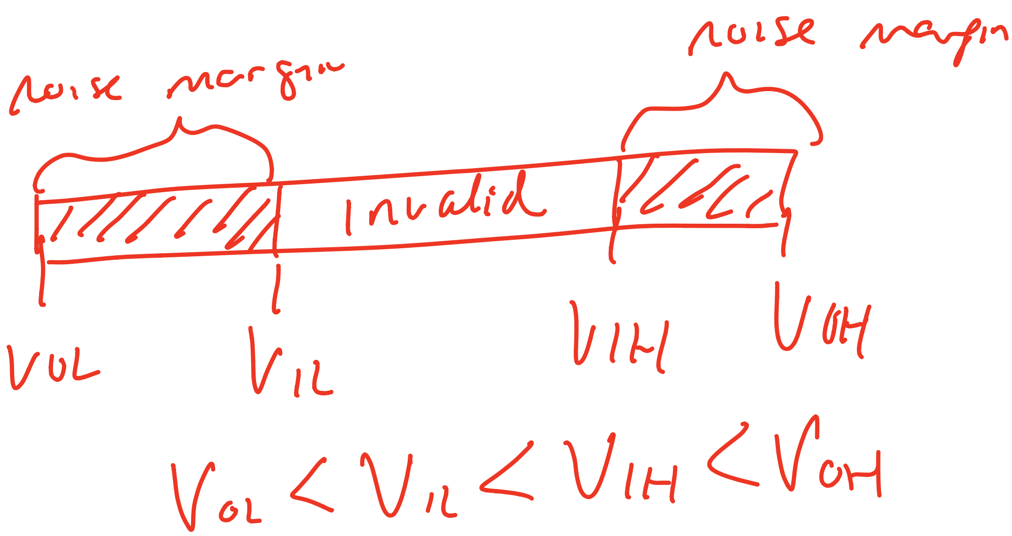
6.004 Tutorial Problems
L05 – The Digital Abstraction

Note: A small subset of essential problems are marked with a red star (★). We especially encourage you to try these out before recitation.

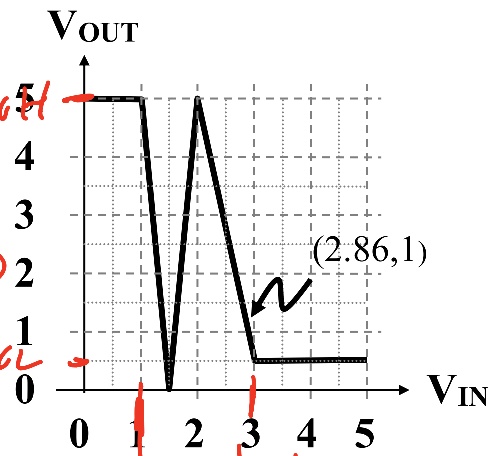




Problem 1. ★

Ms. Anna Logge, founder at a local MIT startup, has developed a device to be used as an inverter. Anna is considering the choice of parameters by which her logic family will represent logic values and needs your help.

The figure on the right shows the voltage transfer curve of a proposed inverter for a new logic family (you can find spare copies below).

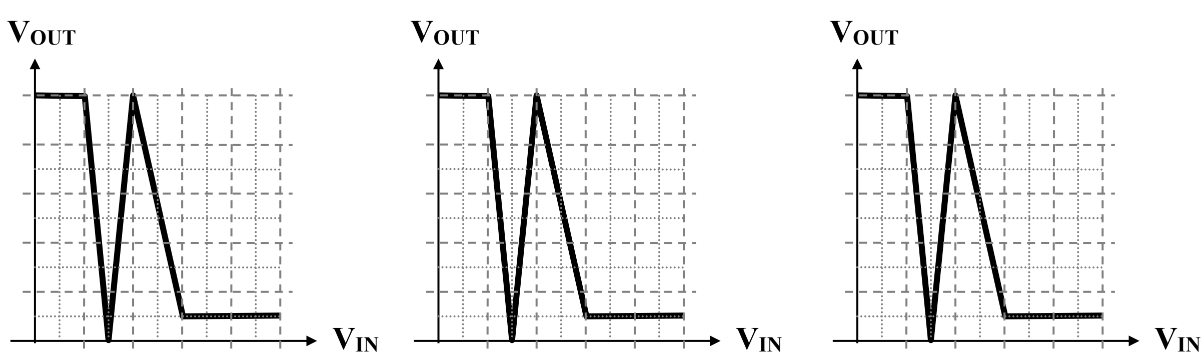


Several possible schemes for mapping logic values to voltages are being considered, as summarized in the incomplete table below. ***Noise Immunity* (the last row) is defined as the smaller of the two noise margins.**

Complete the table by filling in missing entries. Choose each value to maximize the noise margins of the corresponding scheme. **If the numbers in a scheme can't be completed such that the device functions as an inverter with positive noise margins, fill the entries for that column with *X*s.**

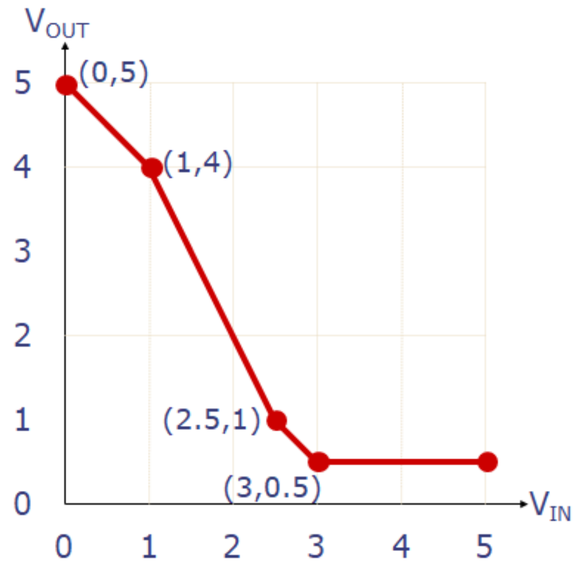
LNI's Possible Logic Mappings:

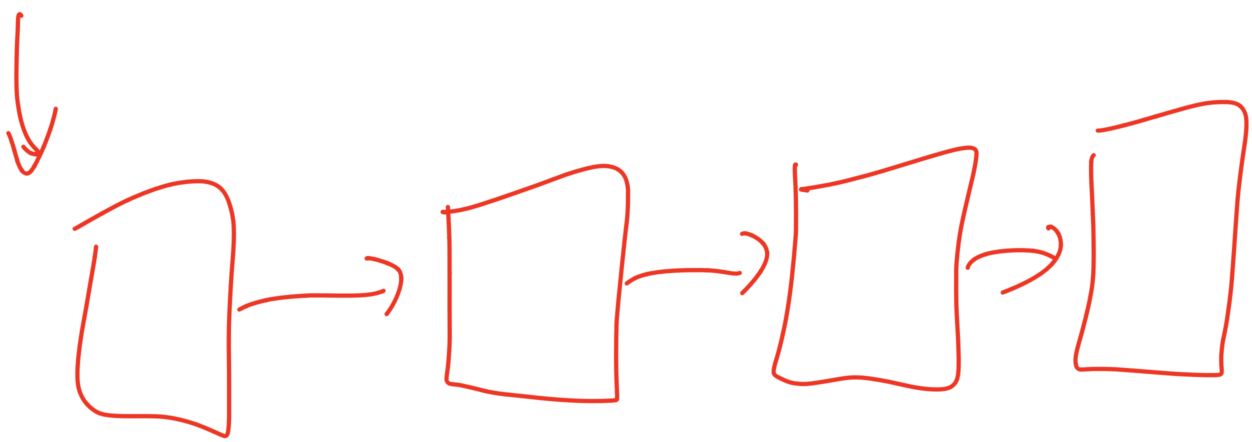
	Scheme A	Scheme B	Scheme C
V_{OL}			1
V_{IL}	2	1	0.5
V_{IH}		3	
V_{OH}			
Noise Immunity			



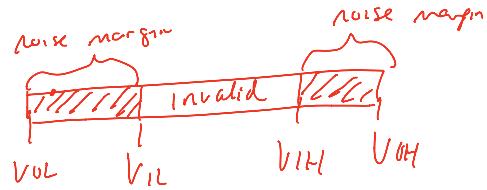
Problem 2.

Suppose that you measured the voltage transfer curve of the device shown below. Can we find a signaling specification (V_{IL}, V_{IH}, V_{OL}, V_{OH}) that would allow this device to be a digital inverter? If so, give the specification that maximizes noise margin.





V_{OL} + noise margin = V_{IL}



$V_{OL} < V_{IL} < V_{IH} < V_{OH}$

V_{OH} − noise = V_{IH}

Problem 3. ★

Suppose we define all signaling thresholds in our digital system to be relative to the supply voltage, V_{DD}:

- $V_{OL} = 0.1V_{DD}$
- $V_{IL} = 0.4V_{DD}$
- $V_{IH} = 0.6V_{DD}$
- $V_{OH} = 0.9V_{DD}$

We want to connect two types of digital devices, A and B, that use different supply voltages, $V_{DD,A}$ and $V_{DD,B}$. **We are given that $V_{DD,A}$ = 1V**.

(1) In the circuit below, under what range of supply voltages $V_{DD,B}$ will the system work correctly?

A → B

A→B:

① $V_{OL,A} \le V_{IL,B}$

$0.1 V_{DD,A} \le 0.4 V_{DD,B}$

$\frac{0.1}{0.4} \le V_{DD,B}$

$0.25 \le V_{DD,B}$

② $V_{OH,A} \ge V_{IH,B}$

$0.9 V_{DD,A} \ge 0.6 V_{DD,B}$

$0.9/0.6 \ge V_{DD,B}$

$1.5 \ge V_{DD,B}$

(2) In the circuit below, under what range of supply voltages $V_{DD,B}$ will the system work correctly?

A → B → A

B→A:

③ $V_{OL,B} \le V_{IL,A}$

$0.1 V_{DD,B} \le 0.4 V_{DD,A}$

$V_{DD,B} \le 4$

④ $V_{OH,B} \ge V_{IH,A}$

$0.9 V_{DD,B} \ge 0.6 V_{DD,A}$

$V_{DD,B} \ge 2/3$

$\frac{2}{3} \le V_{DD,B} \le 1.5$

(3) For the same circuit as in part 2, under what range of supply voltages $V_{DD,B}$ will the system have noise margins of at least 0.1V?

A → B

① $V_{OL,A} + 0.1 \le V_{IL,B}$

② $V_{OH,A} - 0.1 \ge V_{IH,B}$

B → A

③ $V_{OL,B} + 0.1 \le V_{IL,A}$

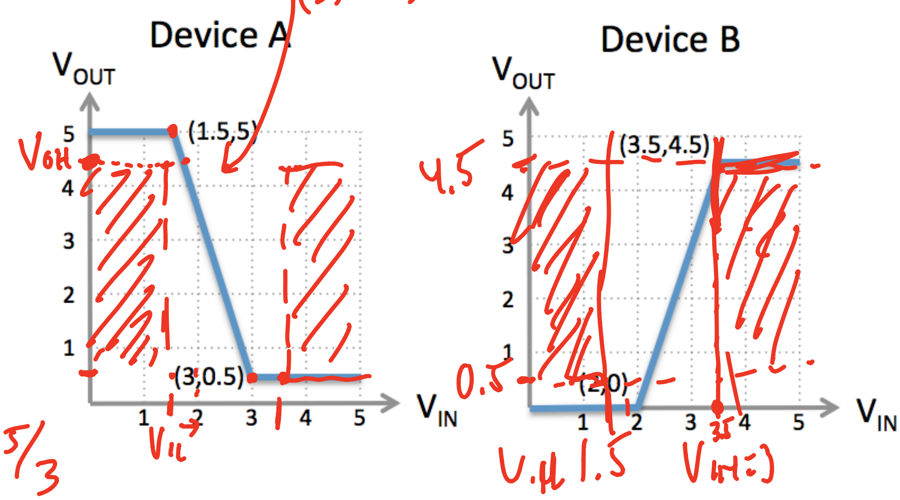
④ $V_{OH,B} - 0.1 \ge V_{IH,A}$

noise margin

noise margin

Problem 4. ★

The following are voltage transfer characteristics of single-input, single-output devices to be used in a new logic family:



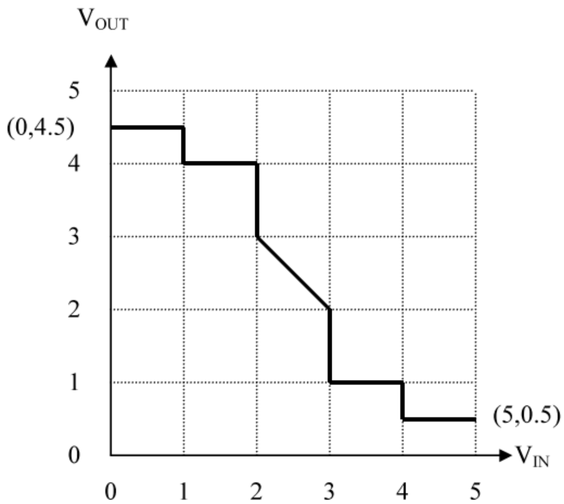
Your job is to choose a single set of signaling thresholds V_{OL}, V_{IL}, V_{OH}, and V_{IH} to be used with both devices to give the best noise margins you can. Recall that the VTC can touch the edge of the forbidden regions but not pass through those regions. Fill in your answers below, together with the resulting noise margins. You'll get partial credit for anything that works with nonzero noise margins; for full credit, maximize the noise immunity (i.e., the smaller of the two noise margins).

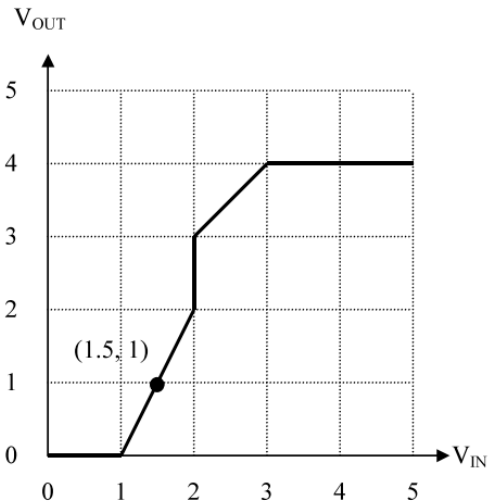
V_{OL} = ______ V_{IL} = ______ V_{IH} = ______ V_{OH} = ______

Low Noise Margin = ______ **High Noise Margin** = ______

Problem 5.

The following are voltage transfer characteristics of devices to be used in a new logic family as an inverter and buffer, respectively:



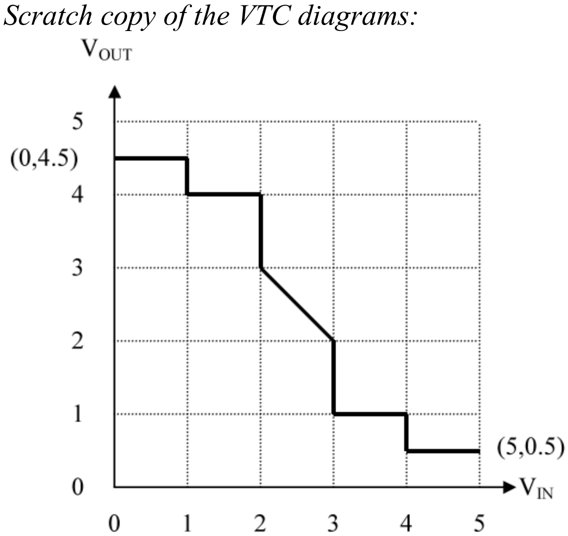


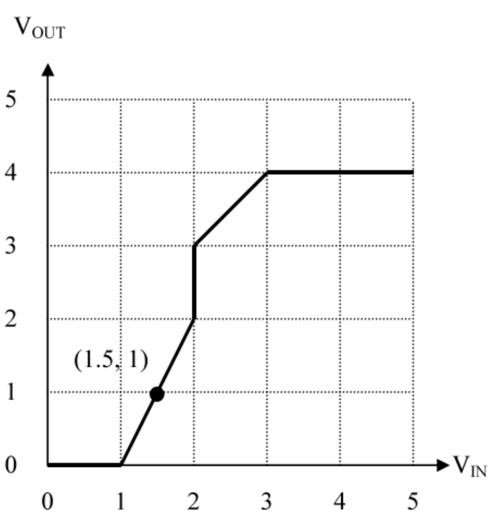
Your job is to choose a single set of signaling thresholds V_{OL}, V_{IL}, V_{OH}, and V_{IH} to be used with both devices to give the best noise margins you can. Recall that the VTC can touch the edge of the forbidden regions but not pass through those regions. Fill in your answers below, together with the resulting noise margins. You'll get partial credit for anything that works with nonzero noise margins; for full credit, maximize each of the noise margins.

V_{OL} = ______ V_{IL} = ______ V_{IH} = ______ V_{OH} = ______

Low Noise Margin = ______ High Noise Margin = ______

Scratch copy of the VTC diagrams:

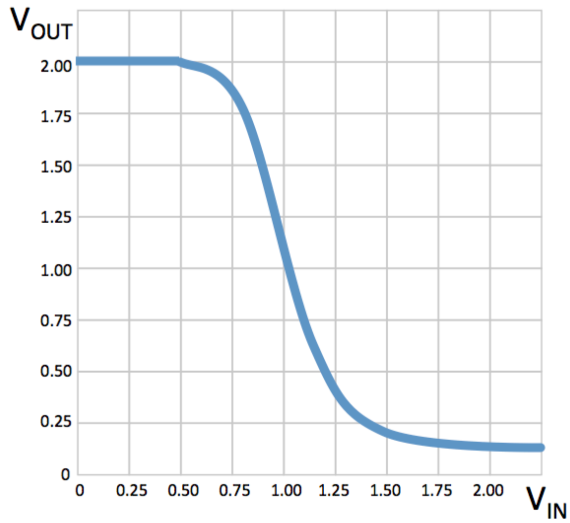




Problem 6. ★

The voltage transfer curve for an inverter is shown to the right. The manufacturer decided to crowdsource the digital signaling specifications for their inverter and has received some suggestions for V_{OL}, V_{IL}, V_{IH}, and V_{OH}, presented in tabular form below.

For each suggested specification, determine if the inverter would be a legitimate combinational device with non-zero positive noise margins. If it is a legitimate combinational device, give the noise immunity of the inverter (the smaller of the low and high noise margins) when operating under that specification. If the inverter wouldn't be a legitimate combinational device, please write NOT LEGIT in the rightmost column.

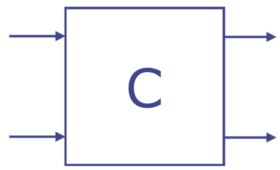


Fill in rightmost column for each suggested specification.

Suggestion	V_{OL}	V_{IL}	V_{IH}	V_{OH}	*Noise immunity, or NOT LEGIT*
#1	0.00	0.50	1.50	2.00	
#2	0.25	0.75	1.25	1.75	
#3	0.50	0.75	1.25	1.50	
#4	0.75	0.50	1.75	1.50	

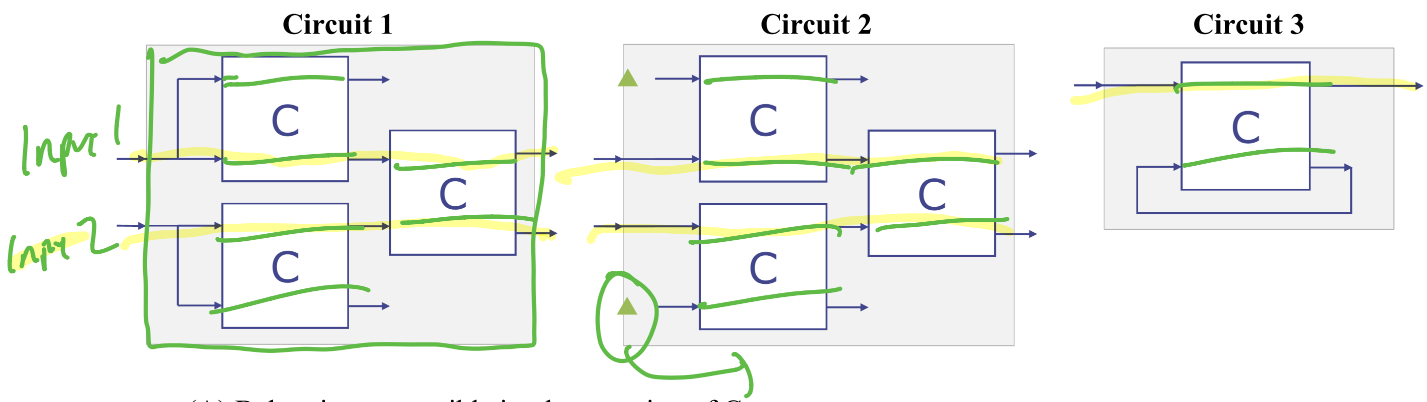
Problem 7. ★

The circuit C shown below is a 2-input, 2-output combinational device.

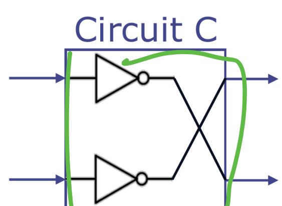


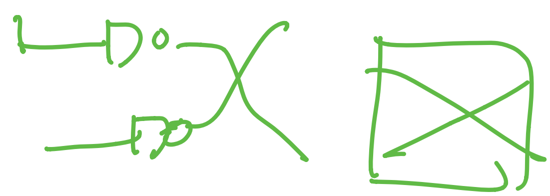
Condition for combinational:
(1) not have mem
(2) output ⇒ function of input

Each of the three circuits below contains multiple copies of circuit C. **Note**: the ▲ symbol indicates a "floating" input.



(A) Below is one possible implementation of C:

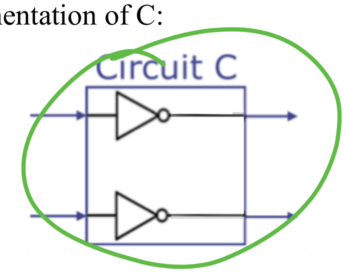


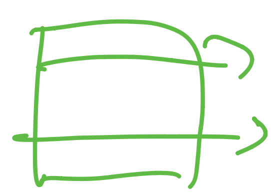


Given this implementation, determine which of the above circuits are combinational.

Circuit 1 __yes__ **Circuit 2** __N__ **Circuit 3** __Y__

(B) Below is an alternative implementation of C:





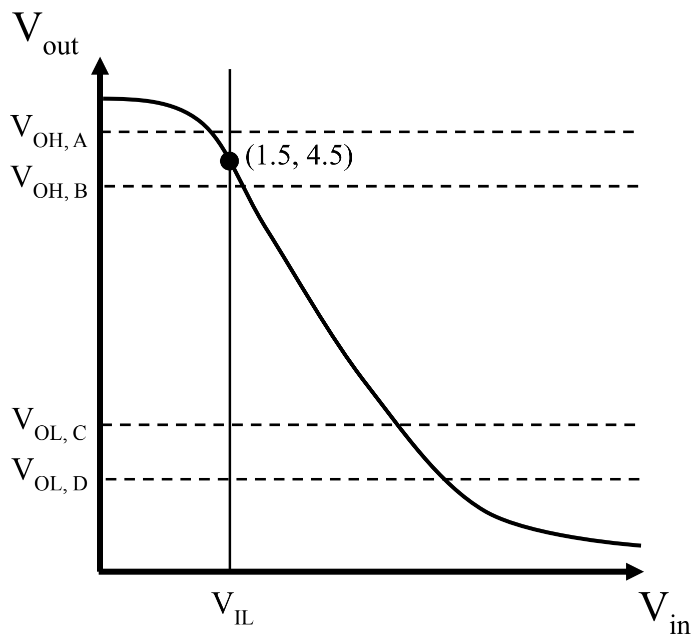
Given this alternative implementation, determine which of the above circuits are combinational.

Circuit 1 __Y__ **Circuit 2** __Y__ **Circuit 3** __Y__

From past quizzes:

Problem 8. Static Discipline (13 points)

Consider the voltage transfer characteristic (VTC) of a hypothetical Device F shown below (not to scale). Ben Bitdiddle is considering two different choices for each of V_{OH} and V_{OL}, as indicated by the dashed lines on the graph below. The precise values of these thresholds are listed in the table. V_{IL} is known to be 1.5V.



Variable	Value (V)
$V_{OH,A}$	4.7
$V_{OH,B}$	4.2
$V_{OL,C}$	1.8
$V_{OL,D}$	1.0
V_{IL}	1.5

(A) (4 points) V_{IH} is not known right now but is guaranteed to be between 3.0 V and 4.0 V. For each choice of V_{OH}, circle **YES** if it both follows the static discipline and has a positive **high noise margin**, or circle **NO** if it does not satisfy both of these conditions. If the answer depends on knowing values that are not provided, then circle **DON'T KNOW**.

$V_{OH, A}$ (circle one): **YES** **NO** **DON'T KNOW**

$V_{OH, B}$ (circle one): **YES** **NO** **DON'T KNOW**

(B) (4 points) For each choice of V_{OL}, circle **YES** if it both follows the static discipline and allows for a positive **low noise margin**, or circle **NO** if it does not satisfy both of these conditions. If the answer depends on knowing values that are not provided, then circle **DON'T KNOW**.

$V_{OL, C}$ (circle one): **YES** **NO** **DON'T KNOW**

$V_{OL, D}$ (circle one): **YES** **NO** **DON'T KNOW**

(C) (2 points) Suppose Ben settles on the following signaling specification which follows the static discipline:

$$V_{IL} = 1.5\text{ V}$$
$$V_{IH} = 3.8\text{ V}$$
$$V_{OL} = 1.4\text{ V}$$
$$V_{OH} = 4\text{ V}$$

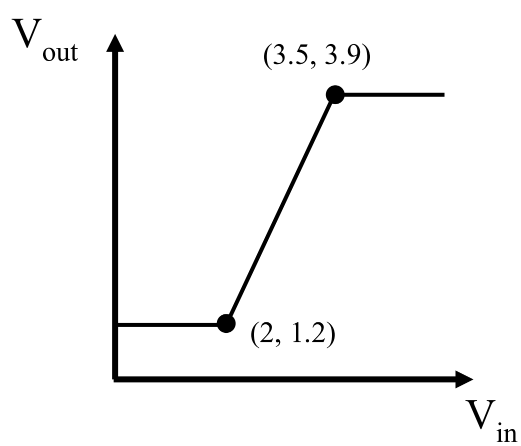
Calculate the high and low noise margins as well as the noise margins of the device as a whole.

High Noise Margin: ____________________ V

Low Noise Margin: ____________________ V

Overall Noise Margin: ___________________ V

(D) (3 points) Alyssa P. Hacker has another Device G with the following VTC that she would like to use with Device F. Can she use Device G with Device F and the signaling specifications described in part (C)? If she can, circle **NO CHANGES**. Otherwise, circle **CHANGES NEEDED** and change **exactly one** of the thresholds of the signaling specification to a new value such that Device G can be used while obeying the static discipline and maximizing noise margins. Also calculate the overall noise margin of the resulting system, regardless of whether or not you changed any thresholds. Keep in mind that VTCs may touch the edge of, but not enter the forbidden region.



Are changes needed? (circle one): NO CHANGES CHANGES NEEDED

Threshold to Change (circle one): V_{IL} V_{IH} V_{OL} V_{OH}

Value to change to: __________________ V

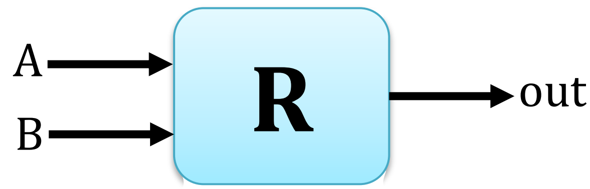
Overall Noise Margin: __________________ V

From past quizzes:

Problem 9. Static Discipline (13 points)

The R module below outputs 0.5V when $\min(V_A, 0.5V_B) > 2V$ for 25ns and outputs 6V when $\min(V_A, 0.5V_B) < 1.5V$ for 25ns. This is summarized in the equation below (assume all voltages are positive). Also, assume this circuit obeys a digital signaling specification where low voltages correspond to digital 0 and high voltages correspond to digital 1.

$$V_{out} = \begin{cases} 0.5V, & \min(V_A, 0.5 * V_B) > 2V \\ 6V, & \min(V_A, 0.5 * V_B) < 1.5V \\ 0 \leq ??? \leq 6V, & otherwise \end{cases}$$



(A) (2 points) If we apply constant V_A, V_B for 25ns and then measure $\boldsymbol{V_{out} = 0.5V}$, what can we conclude about V_B?

C1: $V_B < 3V$

C2: $V_B \leq 4V$

C3: $V_B > 4V$

C4: $V_B \geq 3V$

C5: None of the above

(label: 6A) Best conclusion about V_B (Select one): C1 … C2 … C3 … C4 … C5

(B) (3 points). What Boolean expression does R implement? Specify an equation using A and B.

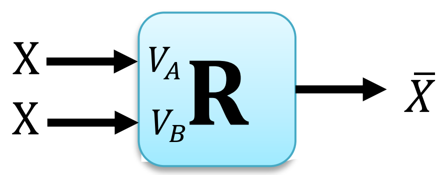
(label: 6B) Boolean Expression: out(A, B) = ________________

(C) (5 points) Find a signaling specification that maximizes noise immunity for the R module.

(label: 6C_1) V_{OL} = ____ , V_{IL} = _____ , V_{IH} = _____ , V_{OH} = ______

(label: 6C_2) Noise Immunity = ________________

(D) (3 points) Suppose one wishes to using the R module as an inverter, as shown below. What is the noise immunity of this device?



(label: 6D) Noise Immunity: _______________