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$10^0 = 2^0 = 16^0 = 1$

Decimal

$\underset{10^2}{2}\,\underset{10^1}{1}\,\underset{10^0}{3} = 2\times10^2 + 1\times10^1 + 3\times10^0$

$= 200 + 10 + 3 = 213$

Binary

$\underset{2^3}{1}\,\underset{2^2}{0}\,\underset{2^1}{1}\,\underset{2^0}{1}$

$= 1\times2^3 + 0\times2^2 + 1\times2^1 + 1\times2^0$

$= 8 + 0 + 2 + 1 = 11$

Hexademical

0x9E

10	11	12	13	14	15
A	B	C	D	E	F

$9\times16^1 + 14\times16^0$

$= 9\times16 + 14$

$= 158$

Modular Arithmetic

CIRCLES

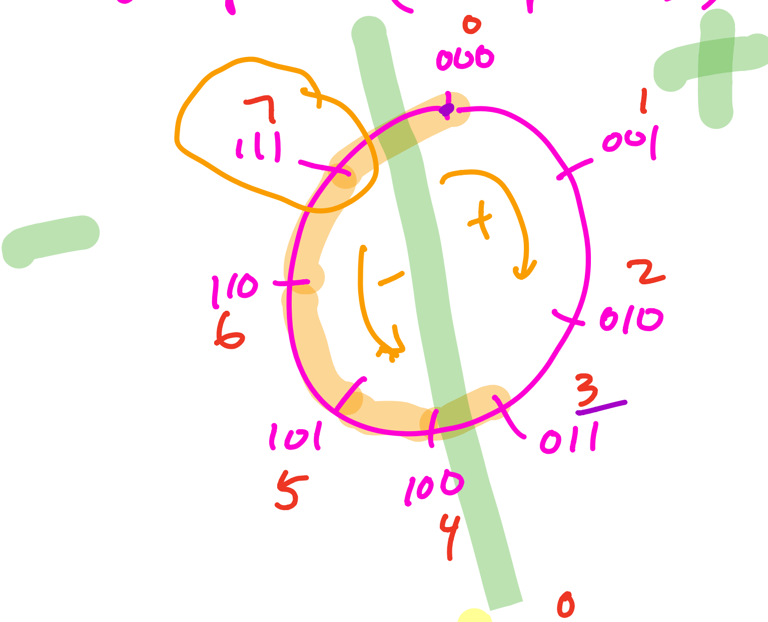
Unsigned (all pos #s)

$0 \to 7$

$0 \to (2^3-1)$

$0 \to 2^N-1$

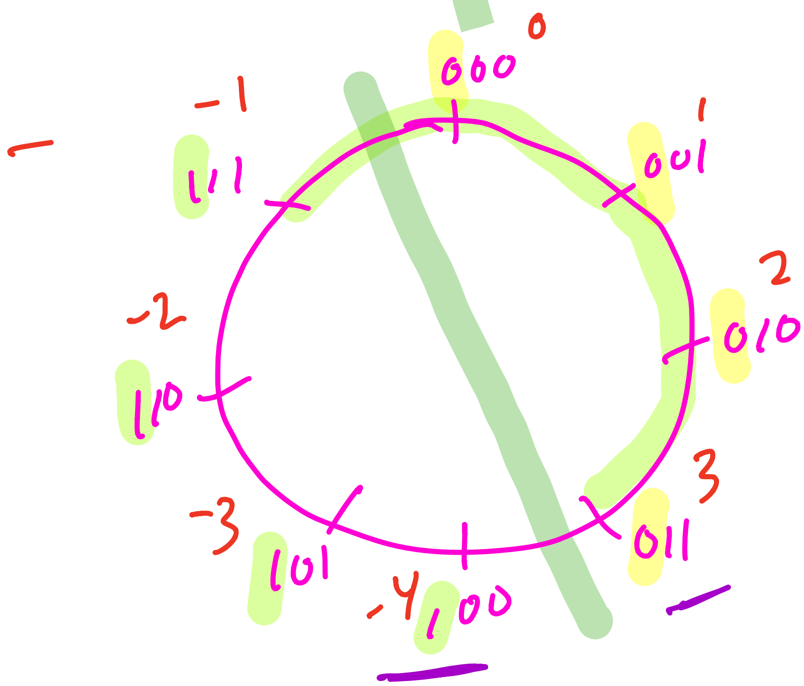
3 digits



we're working 3 digits

$(4-5) \bmod 2^3$

$= 111$ aka 7



+ Signed

$3-4=-1$

$-4 \to 3$

$\frac{2^3}{2} = 2^2$ aka 2^{N-1}

$-2^{N-1} \to 2^{N-1}-1$

what is range of #s we can in a signed w/ 5 digit binary

e.g. 3 digit binary

$-4 \to 3$

$-16 \to 15$

$$-2^{5-1} = -2^4 = -16$$

$$2^{N-1} - 1 = 2^4 - 1 = 15$$

what is range of #s we can in a unsigned w/ 5 digit binary

e.g. for 3 digit

$$0 \to 2^N - 1 = 0 \to 2^3 - 1$$

$0 \to 7$

$0 \to 31$

$0 \to 31$

$$2^5 - 1 = 32 - 1$$

Two Complement Arithmetic

Convert # from pos → neg
neg → pos

☆ $(-x) + x = 0$

$01100101 = 1\times2^6 + 1\times2^5 + 0 + 0 + 1\times2^2 + 0 + 1 =$
$2^6\ 2^5\ 2^4\ 2^3\ 2^2\ 2^1\ 2^0$

$64+32+4+1 = 101$

① flip every digit

10011010

② add 1

10011011

$= -101$

$$\begin{array}{r} 1\,1\,1\,1\,1\,1\,1\;\; \\ 0\,1\,1\,0\,0\,1\,0\,1 \\ +\;1\,0\,0\,1\,1\,0\,1\,1 \\ \hline 0\,0\,0\,0\,0\,0\,0\,0 \end{array}$$

$-A + A = 0 = -1 + 1$

$-A = \underbrace{(-1 - A)}_{1\ 11111} + 1$

$$\begin{array}{r} 1\ 11111 \\ -\ A_{n-1} \quad A_0 \\ \hline \overline{A_{n-1}} \ldots \overline{A_1}\,\overline{A_0} \end{array}$$

flipping

$219 = 128+64+16+8+2+1$

$= \underbrace{0b1101}_{D}\_\underbrace{1011}_{B} \Rightarrow 0xDB$

1011 B
1100 C
1101 D
1110 E
1111 F

6.004 Tutorial Problems
L01 – Binary Encoding and Arithmetic

Note: A subset of essential problems are marked with a red star (★). We especially encourage you to try these out before recitation.

$21 - 16 = 5$
$5 - 4 = 1$

Problem 1. Encoding positive integers

1. What is the 5-bit binary representation of the decimal number 21? ★

2^4	2^3	2^2	2^1	2^0
16	8	4	2	1
1	0	1	0	1

$\boxed{0b10101}$

2. What is the hexadecimal representation for decimal 219 encoded as an 8-bit binary number?

10	11	12	13	14	15
A	B	C	D	E	F

$\cancel{16^2}$ 16^1 16^0

$\cancel{256}$ 16 1

13 11

$\boxed{0xDB}$

$219 - 208 = 011$

3. What is the hexadecimal representation for decimal 51 encoded as a 6-bit binary number?

4. The hexadecimal representation for an 8-bit unsigned binary number is 0x9E. What is its decimal representation? ★

5. What is the range of integers that can be represented with a single unsigned 8-bit quantity?

6. Since the start of official pitching statistics in 1988, the highest number of pitches in a single game has been 172. Assuming that remains the upper bound on pitch count, how many bits would we need to record the pitch count for each game as an unsigned binary number?

7. Compute the sum of these two 4-bit unsigned binary numbers: 0b1101 + 0b0110. Express the result in hexadecimal. ★

$$\begin{array}{r} 1101 \\ +\underline{0110} \end{array}$$

① Find binary 21 ② flip all digit ③ add 1

first 0 pos

first 1 negative

Problem 2. Two's complement representation → Signed

1. What is the 6-bit two's complement representation of the decimal number -21?

21 = 16 + 4 + 1

0b 010101 flip 0b 101010 add 1 0b 101011

2. What is the hexadecimal representation for decimal -51 encoded as an 8-bit two's complement number?

1011	B
1100	C
1101	D
1110	E
1111	F

1) convert hex to binary 2) convert binary to decimal

3. The hexadecimal representation for an 8-bit two's complement number is 0xD6. What is its decimal representation?

0xD6 = 0b 1101_0110

① flip digits, add 1

0b 0010_1001 + 1 = 0b 0010_1010 => -42

1x32 + 1x8 + 1x2 = 42

② 1x-128 + 1x64 (2^7) + 0x32 + 1x16 + 1x4 + 1x2 + 0x1 = -42

4. Using a 5-bit two's complement representation, what is the range of integers that can be represented with a single 5-bit quantity?

5. Can the value of the sum of two 2's complement numbers 0xB3 + 0x47 be represented using an 8-bit 2's complement representation? If so, what is the sum in hex? If not, write NO. ★

6. Can the value of the sum of two 2's complement numbers 0xB3 + 0xB1 be represented using an 8-bit 2's complement representation? If so, what is the sum in hex? If not, write NO. ★

7. Please compute the value of the expression 0xBB – 8 using 8-bit two's complement arithmetic and give the result in decimal (base 10).

8. Consider the following subtraction problem where the operands are 5-bit two's complement numbers. Compute the result and give the answer as a decimal (base 10) number. ★

$$\begin{array}{r} 10101 \\ -\underline{00011} \end{array}$$

Problem 3. Multiples of 4

1. Given an unsigned n-bit binary integer $\boldsymbol{v = b_{n-1} \ldots b_1 b_0}$, prove that $\boldsymbol{v}$ is a multiple of 4 if and only if $\boldsymbol{b_0 = 0}$ and $\boldsymbol{b_1 = 0}$.

2. Does the same relation hold for two's complement encoding?

Problem 4. Encoding text

There are multiple standards to encode characters and strings using binary values. ASCII is a classic standard to encode English alphabet characters (modern formats like UTF support other alphabets, but are typically based on ASCII). ASCII encodes each character using an 8-bit (1-byte) value. The table below shows ASCII's mapping of characters to values.

ASCII (1977/1986), `adapted from https://en.wikipedia.org/wiki/ASCII`

	_0	_1	_2	_3	_4	_5	_6	_7	_8	_9	_A	_B	_C	_D	_E	_F
0_	NUL 0x00	SOH 0x01	STX 0x02	ETX 0x03	EOT 0x04	ENQ 0x05	ACK 0x06	BEL 0x07	BS 0x08	HT 0x09	LF 0x0A	VT 0x0B	FF 0x0C	CR 0x0D	SO 0x0E	SI 0x0F
1_	DLE 0x10	DC1 0x11	DC2 0x12	DC3 0x13	DC4 0x14	NAK 0x15	SYN 0x16	ETB 0x17	CAN 0x18	EM 0x19	SUB 0x1A	ESC 0x1B	FS 0x1C	GS 0x1D	RS 0x1E	US 0x1F
2_	space 0x20	! 0x21	" 0x22	# 0x23	$ 0x24	% 0x25	& 0x26	' 0x27	(0x28	) 0x29	* 0x2A	+ 0x2B	, 0x2C	- 0x2D	. 0x2E	/ 0x2F
3_	0 0x30	1 0x31	2 0x32	3 0x33	4 0x34	5 0x35	6 0x36	7 0x37	8 0x38	9 0x39	: 0x3A	; 0x3B	< 0x3C	= 0x3D	> 0x3E	? 0x3F
4_	@ 0x40	A 0x41	B 0x42	C 0x43	D 0x44	E 0x45	F 0x46	G 0x47	H 0x48	I 0x49	J 0x4A	K 0x4B	L 0x4C	M 0x4D	N 0x4E	O 0x4F
5_	P 0x50	Q 0x51	R 0x52	S 0x53	T 0x54	U 0x55	V 0x56	W 0x57	X 0x58	Y 0x59	Z 0x5A	[0x5B	\ 0x5C	] 0x5D	^ 0x5E	_ 0x5F
6_	` 0x60	a 0x61	b 0x62	c 0x63	d 0x64	e 0x65	f 0x66	g 0x67	h 0x68	i 0x69	j 0x6A	k 0x6B	l 0x6C	m 0x6D	n 0x6E	o 0x6F
7_	p 0x70	q 0x71	r 0x72	s 0x73	t 0x74	u 0x75	v 0x76	w 0x77	x 0x78	y 0x79	z 0x7A	{ 0x7B	| 0x7C	} 0x7D	~ 0x7E	DEL 0x7F

Letter Number Punctuation Symbol Other/non-printable

Computers often store variable-length text as a null-terminated string: a sequence of bytes, where each byte denotes a different character, terminated by the value 0x00 (null) to denote the end of the string. For example, the string "6.004" is encoded as the 6-byte sequence 0x36 0x2E 0x30 0x30 0x34 0x00. For brevity, we can also just stick these hex values together to form one large hex number: 0x362E30303400.

1. Encode your name as a null-terminated ASCII string (use the best approximation if your name contains non-English characters)

2. Decode the following null-terminated ASCII string:

0x 52 49 53 43 2D 56 20 69 73 20 63 6F 6D 69 6E 67 21 00

Problem 5. From Past Quizzes

(A) (2 points) What is the maximum decimal value that can be represented in **7-bit unsigned binary**? What is the minimum decimal value that can be represented in **6-bit 2's complement**?

Largest 7-bit unsigned binary (in decimal):__________________

Smallest 6-bit 2's complement number (in decimal):___________________

(B) (4 points) What is -25 in 7-bit 2's complement encoding? What is -40 in 7-bit 2's complement encoding? Show how to compute -25-40 using 2's complement addition. Is it possible to represent the result in 7-bit 2's complement encoding? If so, **show your binary addition work and write the result in binary**. If not, write **"Not Possible"** and **explain why it's not possible.**

-25 in 7-bit 2's complement notation (0b):_____________________

-40 in 7-bit 2's complement notation (0b):_____________________

-25 -40 in 7-bit 2's complement notation or "Not Possible" (show your work)

(0b):_____________________