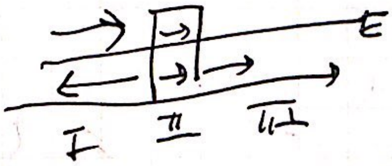
Lecture 9 9/25

$$T = \frac{4(k\gamma)^2}{(\gamma^2+k^2)^2 \sinh^2(\gamma L) + 4\gamma^2 k^2}$$

Recall $k = \sqrt{\frac{2mE}{\hbar^2}}$ $\gamma = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$

$$T(E) = \frac{4E(V_0-E)}{4E(V_0-E)+V_0^2\sinh^2\left(\pi\sqrt{\frac{V_0-E}{E_L}}\right)} \qquad E_L = \frac{\hbar^2\pi^2}{2mL^2}$$

Tunneling



Match bdy conditions

$x=0$ $\begin{cases} \frac{1}{t}+\frac{r}{t} = \frac{a}{t}+\frac{b}{t} \\ \frac{(1-r)}{t} = \frac{\gamma}{ik}\left(\frac{a}{t}-\frac{b}{t}\right) \end{cases}$

$\frac{a}{t} = \frac{1}{2}e^{-\gamma L}\left(1+\frac{ik}{\gamma}\right)e^{ikL}$

$\frac{b}{t} = \frac{1}{2}e^{\gamma L}\left(1-\frac{ik}{\gamma}\right)e^{ikL}$

$\psi_I(x) = e^{ikx} + re^{-ikx}$

$\psi_{II}(x) = ae$

$\frac{2}{t} = \left(1+\frac{\gamma}{ik}\right)\frac{a}{t} + \left(1-\frac{\gamma}{ik}\right)\frac{b}{t}$

$x=L$ $\begin{cases} \frac{a}{t}e^{\gamma L} + \frac{be^{-\gamma L}}{t} = e^{ikL} \\ \frac{ae^{\gamma L}}{t} - \frac{be^{-\gamma L}}{t} = \frac{ike^{ikL}}{\gamma} \end{cases}$

$$t = \frac{4ik\gamma e^{-ikL}}{(\gamma+ik)^2e^{-\gamma L} - (\gamma-ik)^2e^{\gamma L}}$$

$$T = \frac{4(k\gamma)^2}{(\gamma^2+k^2)^2\sinh^2(\gamma L) + 4\gamma^2k^2} = |t|^2$$

Recall $k = \sqrt{\frac{2mE}{\hbar^2}}$ $\gamma = \sqrt{\frac{2m(V_0-E)}{\hbar^2}}$

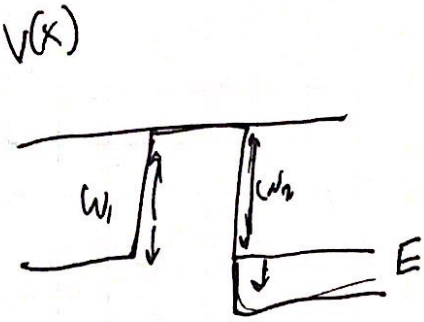
$\sinh x = \frac{e^x - e^{-x}}{2} \xrightarrow[x\to\infty]{} \frac{e^x}{2}$

$$T(E) = \frac{4E(V_0-E)}{4E(V_0-E)+V_0^2\sinh^2\left(\pi\sqrt{\frac{V_0-E}{E_L}}\right)}$$

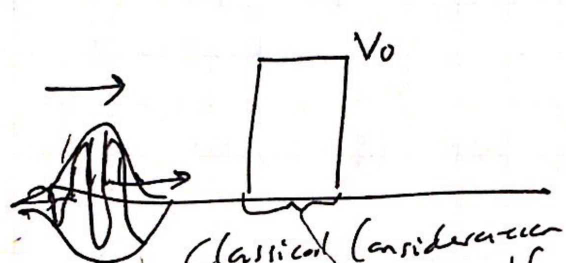
$$T \to \frac{16E(V_0-E)}{V^2}e^{-2\gamma L}$$

$E_L = \frac{\hbar^2\pi^2}{2mL^2}$ $\longrightarrow e^{-2\gamma L} = e^{-2G}$ $G = \gamma L$

Something more ~~along~~ along, greater current?



Another Issue



barrier

Classical Consideration

$V_g = \frac{d\omega}{dk} = \frac{\hbar k_0}{m}$

If it tunnels through same speed

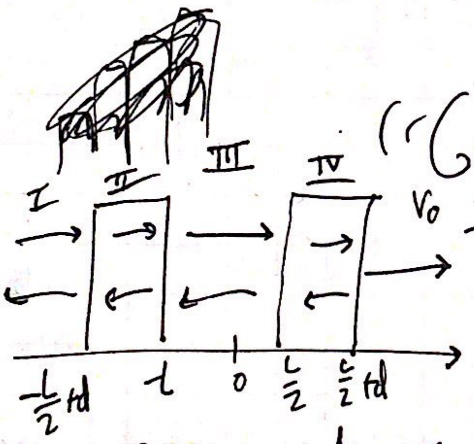
how long is the particle gonna take?

$$\Psi(x,t) = \sum_j a_j \phi_j(x) e^{-iE_j t/\hbar}$$

Observation

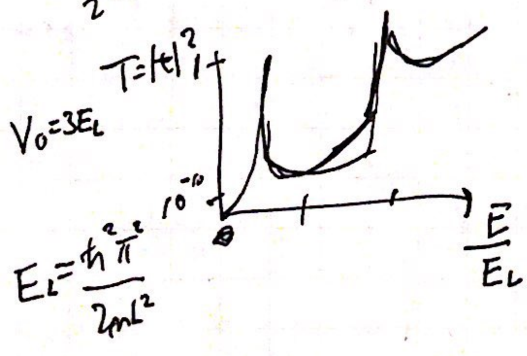
The particle goes through boundary instantaneously & goes out the other side.

"Glue barriers together"

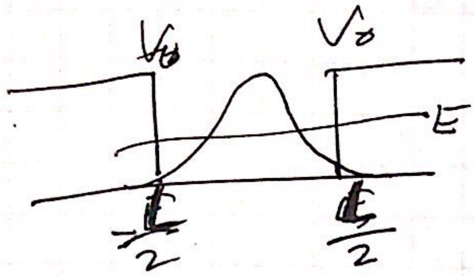


$$\Psi_I(x) = e^{ikx} + re^{-ikx}$$
$$\Psi_{II}(x) = ae^{\alpha x} + be^{-\alpha x}$$
$$\Psi_V(x) = te^{ikx}$$

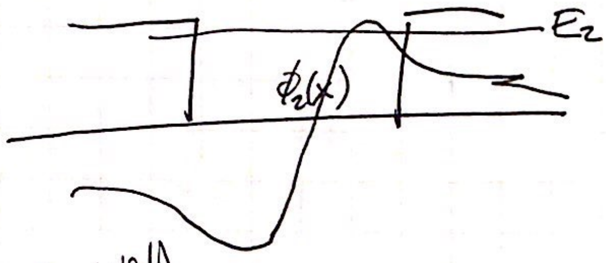
can solve for transmission & reflection coefficients



"Why would we get a baseline that looks like this?"



"like a resonator"



Applications:

semiconductor system

barriers using diff. alloy

carriers w/ voltage across them.

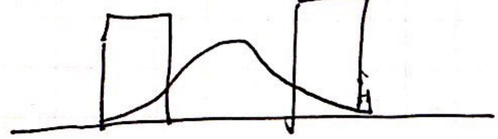
Gets current massive.

resonate tunnel diode

↳ not as effective because 3D only for certain sub energies.

Leaky well

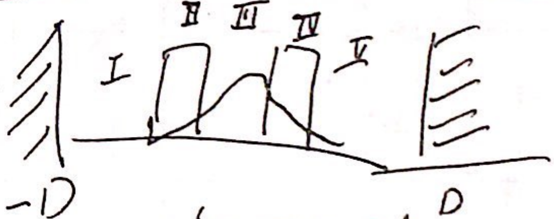
$\Psi(x, t=0)$



populate ground state of well.

"leaking out going out" so amplitude is going down.

$$\Psi(x,t) = \sum_j a_i \phi_i(x) e^{-iE_j t/\hbar}$$



put barriers at end.
discrete version, easier for computer

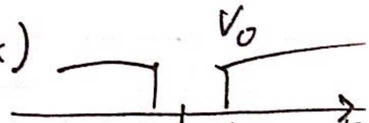
$$\Psi_{III}(x,t) = \cos(kx)$$

$$\Psi_{IV}(x,t) = ae^{\gamma x} + be^{-\gamma x}$$

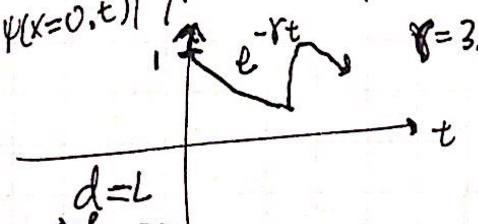
$$\Psi_{V}(x,t) = C\sin(k(x-D))$$

$$\Psi(x,0) = \phi_1(x)$$

of



$|\Psi(x=0,t)|^2 / |\Psi(x=0,t=0)|^2$



$\gamma = 3.876 \times 10^{-5} \frac{E_L}{\hbar}$

$$\Psi(x,t=0) = \sum_j a_j \phi_j(x)$$

$$a_j = \langle \phi_j | \Psi(x,t=0) \rangle$$

$d = L$

$V_0 = 3E_L$

Stuff that leaks out comes back

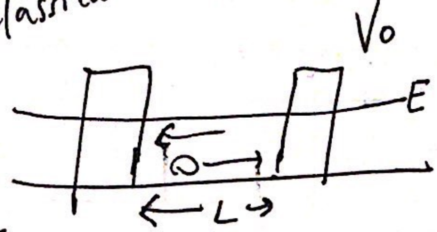
applications of this phenomena:

In nuclear. Quantum wells in semiconductors.

Quantum well: thermal → electric conversion,

Classical analog

Classical analog of physical picture.



$Vt = L$

$\frac{F}{\Delta t} = \frac{V}{L}$

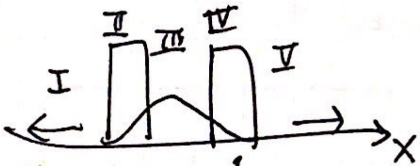
$\gamma = \left(\frac{V}{L}\right) \cdot T$

probability of escaping everytime you hit the wall

$\gamma = \left(\frac{V}{L}\right) e^{-2G}$

$= 2.36 \times 10^{-5} E$ Used for alpha decay represent decay rates.

What would happen if we put together a non-hermatian model



probabability flux?

even $\psi(x) = \psi(-x)$

$\psi_{III}(x) = \cos(kx)$

$\psi_{IV}(x) = a e^{\gamma x} + b e^{-\gamma x}$

$\psi_{V}(x) = t e^{ikx}$ for symmetry

$\psi_{I}(x) = t e^{-ikx}$

Hermitian? ~~Not a~~ ~~hermitian~~

$$\int_{-\infty}^{\infty} (\hat{H}(\psi))^* \hat{Q} \psi \, dx = \int_{-\infty}^{\infty} \psi^* \hat{H} \hat{Q} \psi \, dx$$ hermatian

required for this to be hermatian

subset when $Q = 1$

$$\hat{H} = \frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + V(x)$$

$$\int_{-\infty}^{\infty} (\hat{H}\psi)^* \psi \, dx = \int_{-\infty}^{\infty} \left[\left(\frac{-\hbar^2}{2m} \frac{d^2}{dx^2} + V(x)\right) \psi^*\right] \psi(x) \, dx$$ ...

$$\int_{-\infty}^{\infty} \frac{d^2\phi}{dx^2}\,\phi\, dx = \frac{d\psi^*}{dx}\psi\Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty}\frac{d\psi^*}{dx}\frac{d\psi}{dx}\,dx$$

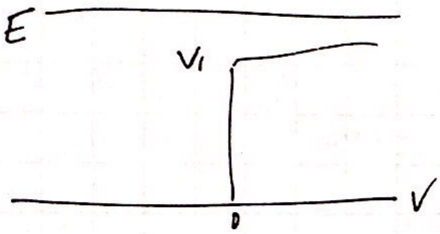
$$= \frac{d\psi^*}{dx}\psi\Big|_{-\infty}^{\infty} - \psi^*\frac{d\psi}{dx}\Big|_{-\infty}^{\infty} + \int_{-\infty}^{\infty}\psi^*\frac{d^2\psi}{dx^2}\,dx$$

$$\int_{-\infty}^{\infty}\Big(\underbrace{\Big(\frac{-\hbar^2}{2m}\frac{d^2}{dx^2}+V(x)\Big)\psi^*(x)}_{H\psi^*}\Big)\psi(x)\,dx = -\frac{\hbar^2}{2m}\Big(\frac{d\psi^*}{dx}\psi\Big|_{-\infty}^{\infty}$$

$$-\psi^*\frac{d\psi}{dx} + \int_{-\infty}^{\infty}\psi^*\hat{H}\psi\,dx$$

$$+\,2ik\Big(1-|r|^2+\frac{k_t}{k_i}|t|^2\Big)$$

? don't remember prefactor



$$\psi_I(x) = e^{ik_ix} + re^{-ik_ix}$$
$$\psi_{II}(x) = te^{ik_ix}$$

this discussion is in the notes that are posted

$$R+T=1$$

$$|r|^2 + \frac{k_t}{k_i}|t|^2 = 1$$

$$-\frac{\hbar^2}{2m}\Big(ikt^*t\Big|^{\infty} - -ikt^*t\Big|^{\infty} + \Big|_{-\infty} + \Big|_{-\infty}\Big)$$

Either the flux gets reflected or transmitted

$$\frac{-\hbar^2}{2m}\left(-4k_i|t|^2\right)$$ end up w/ a term that doesn't vanish at the boundaries.

If we have a nonhermitian model, is probability conserved? is energy conserved? No.

Experience: Hagelstein made a model for nonhermitian. Paper was not published. Review says you have to use a non-hermitian for quantum mechanics.

thoughts about having i for energy? we need imaginary part for exponential decay

$$E = 0.525667 - i0.00001937\ldots$$ answer not real?

$$E = Re\{E\} - \frac{i\hbar\gamma}{2}$$

Hermitian $\Psi(x,t) = \psi(x)e^{-iEt/\hbar}$ $\frac{Im\{E\}}{\hbar}$

$P(x,t) = |\psi(x)|^2$

Nonhermitian $\psi(x,t) = \psi(x)e^{-i\frac{Re\{E\}t}{\hbar}}e^{-\frac{\gamma t}{\hbar}}$

$\gamma = 3.87586 \times 10^{5} \dfrac{E_L}{\hbar}$

$$E = \mathrm{Re}\{E\} - \frac{i\hbar\gamma}{2}$$

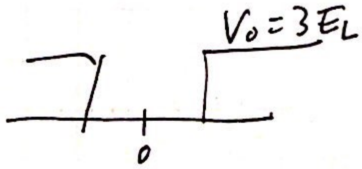
Same as the one from his simulation

the decay rate matches

One conclusion: non-standard, non-hermetian analysis actually works

γ close to potential of infinite walls

0.525683



also binding energy of the ?? state for leaky well.

non-hermatian widely used for things related to decay.