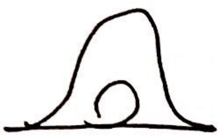
Lecture 4



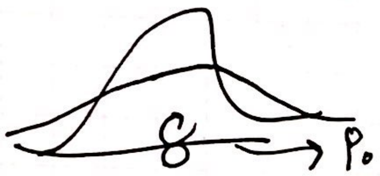
$$\psi(x,0) = e^{-\frac{\pi}{2}x^2}$$

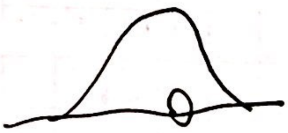
$$\psi(x,0) = \int A(k)\, e^{ikx}\, \frac{dk}{2\pi}$$

$$A(k) = \int \psi(x,0)\, e^{-ikx}\, dx = \sqrt{2}\, e^{-\frac{k^2}{2\pi}}$$

$$\psi(x,t) = \int A(k)\, e^{ikx}\, e^{-\frac{i\hbar k^2 t}{2m}}\, dk$$

$$\psi(x,t) = \frac{1}{\sqrt{1+\frac{i\hbar\pi t}{m}}}\, e^{-\frac{\pi x^2}{2\left(1+\frac{i\hbar t\pi}{m}\right)}}$$





$$\psi(x,0) = e^{-\frac{\pi}{2}x^2}\, e^{ik_0x}$$

$$\hbar k_0 = p_0 = m v_g \Rightarrow v_g = \frac{\hbar k_0}{m}$$

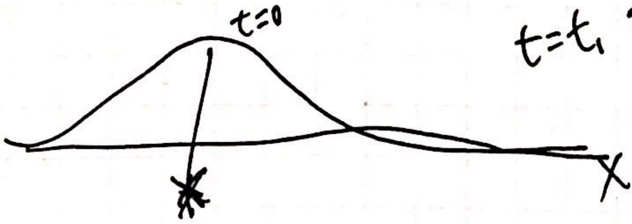
$$A(k) = \int \psi(x,0)\, e^{-ikx}\, dx \Rightarrow A(k-k_0) = \int \psi(x,0)\, e^{ik_0x}\, e^{-ikx}\, dx$$

$$A(k) = \sqrt{2}\ {}^{-\frac{(k-k_0)^2}{2\pi}}$$

$$\psi(x,t) = \int A(k)\, e^{ikx}\, e^{-i\omega t} =$$

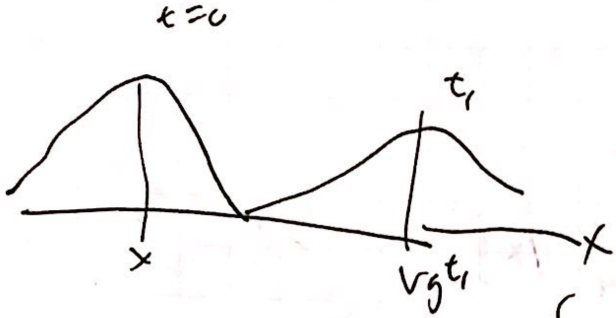
$$\hbar\omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m} \Rightarrow \boxed{\omega = \frac{\hbar k^2}{2m}}$$

$$\psi(x,t) = \frac{1}{\sqrt{1+\frac{i\hbar\pi t}{m}}}\, e^{ik(x-v_g t)}\, e^{-\frac{\pi}{2}\left(1+\frac{i\hbar\pi t}{m}\right)(x-v_g t)^2}$$



$t = t_1$?

amplitude of distribution gets lower



group velocity

$$\frac{d\omega}{dk} = v_g = \frac{\hbar k}{m}$$

$$\langle x \rangle = v_g t$$

$$\psi(x,t) = \int A(k) e^{ikx} e^{-i\omega t} \frac{dk}{2\pi}$$

← time distribution
plane waves pass through optical fibers.

$$\omega(k) = \omega_0 + \frac{d\omega}{dk}\Big|_{k=0} (k-k_1) + \frac{d^2\omega}{2dk^2}(k-k_1)^2 \dots$$

Optics

$$\psi(x,t) = \int A(k) e^{ikx} e^{-i(\omega_0 + \frac{d\omega}{dt}(k-k_1))t}$$

$$\Rightarrow \psi(x,t) = e^{-i\omega_0 t} e^{ik_0 v_g t} \int_{-\infty}^{\infty} A(k) e^{ik(x-v_g t)} \frac{dk}{2\pi}$$

$$P(x,t) = |\psi(x - v_g t)|^2$$

$$\int \psi(x,0) = \int A(k) e^{ikx} \frac{dk}{2\pi}$$

$$\omega(k) = \omega_0 + \frac{dk}{dk}\Big|_{k=0} (k-k_0) + \underbrace{\frac{d^2\omega}{2dk^2}(k-k_0)^2}$$

→ something spreads!

on average things follow newton's law

$$E = \sqrt{c^2p^2 + (m_0c^2)^2} \qquad \hbar\omega = \sqrt{c^2p^2 + (m_0c^2)^2}$$

$$\Rightarrow \omega = \frac{1}{\hbar}\sqrt{c^2\hbar^2k^2 + (m_0c^2)^2}$$

$$\frac{\partial^2\psi}{\partial x^2} = -k^2\psi \qquad \frac{\partial\psi}{\partial t} = i\omega = \frac{-i\hbar k^2}{2m} \Rightarrow \frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} \stackrel{?}{=} \frac{i\hbar\,\partial\psi}{\partial t}$$

$$\boxed{\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + U\psi(x) = \frac{i\hbar\,\partial\psi}{\partial t}} \qquad \Rightarrow \frac{\hbar k^2}{2m}$$

$\nabla\cdot D=0 \qquad \nabla\times\vec{E}=\frac{\partial B}{\partial t}$ ①

$\nabla\cdot B=0 \qquad \nabla\times H=\frac{\partial D}{\partial t}$ ②

$D=\varepsilon_0\vec{E}$

$B=\mu\vec{H}$

$\nabla\times(\nabla\times\vec{E})=-\frac{\partial}{\partial t}\nabla\times\vec{B}=-\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}$

$\nabla\times(\nabla\times\vec{E})=\nabla(\nabla\cdot E)-\nabla^2 E \quad\Rightarrow \nabla^2 E-\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}=0$

$\psi(x,t)=\int A(k)e^{ikx}e^{-i\omega t}\frac{dk}{2\pi}$ $\boxed{c=\frac{1}{\sqrt{\mu_0\varepsilon_0}}}$

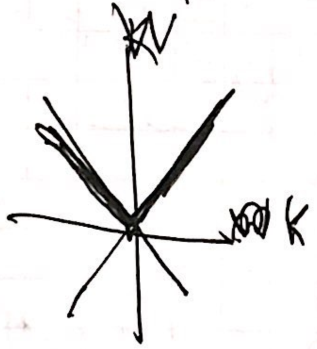
$$\nabla^2 E=\left(\begin{array}{lll}\frac{\partial^2 E_x}{\partial x^2}+\frac{\partial^2 E_x}{\partial x^2}+\frac{\partial^2 E_x}{\partial z^2} \\ \frac{\partial^2 E_y}{\partial x^2} \quad \cdots \quad \frac{\partial^2 E_y}{\partial z^2}\end{array}\right.$$

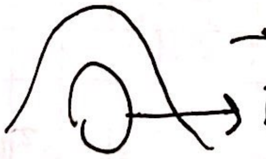
3 coupled relation

$\frac{\partial^2\psi}{\partial x^2}=-k^2\psi=-\frac{1}{c^2}\omega^2\psi$

$k^2=\frac{\omega^2}{c^2} \Rightarrow \boxed{k=\frac{\omega}{c}}$

$=\boxed{\omega=\pm c|k|}$





$$\left\{\begin{array}{l}\frac{dx}{dt}=\frac{p}{m} \\ \frac{dp}{dt}=F\end{array}\right.$$

do you expect to see classical behavior

$\langle x\rangle$

$\langle p\rangle$

$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2}$

$-F_0 x\psi$

$=i\hbar\frac{\partial\psi}{\partial t}$

$e^{i\theta(t)}$ $e^{\frac{ip(t)}{\hbar}}$ $\psi(x,t) = \frac{1}{\sqrt{1+\frac{ik\hbar t}{m}}} e^{i\theta(t)} e^{\frac{ip(t)}{\hbar}(x-\bar{x})} e^{-\frac{k}{2}\frac{(x-\bar{x})^2}{(1+\frac{ik\hbar t}{m})}}$

$$\boxed{\frac{d\bar{x}}{dt} = \frac{p(t)}{m} \qquad \frac{dp(t)}{dt} = F} \qquad \frac{\hbar\, d\theta}{dt} = \frac{-p^2}{2m} - F\bar{x}$$

average satisfies Newton's law

probability follows classical trajectory