6.728 Lecture 32 11/22

From before

$$\boxed{P_1(N,E,V) = \frac{\Omega_1(N_1,E_1,V_1)\, e^{-\frac{(E-\mu N_1+pV_1)}{k_BT}}}{\sum_{N_1'}\sum_{E_1'}\sum_{V_1'} \Omega_1(N_1',E_1',V_1')\, e^{-\frac{(E_1'-\mu N_1'+pV_1)}{k_BT}}}}$$

Example

$$\text{SHO} \quad P_n(T) = \frac{e^{-E_n/k_BT}}{\sum_{n'} e^{-E_{n'}/k_BT}} = \frac{e^{-\frac{\hbar\omega_0}{k_BT}n}}{\sum_{n'} e^{\frac{-\hbar\omega_0 n'}{k_BT}}} \qquad \langle\langle \hat{n} \rangle\rangle = \sum_n n P_n(T) = \frac{1}{e^{\frac{\hbar\omega_0}{k_BT}}-1}$$

$$\langle\langle \hat{H} \rangle\rangle = \hbar\omega_0\left(\langle\langle \hat{n}\rangle\rangle + \frac{1}{2}\right) = \hbar\omega_0\left(\frac{1}{e^{\frac{\hbar\omega_0}{k_BT}}-1} + \frac{1}{2}\right)$$

black body

$$\langle\langle \hat{H} \rangle\rangle = \left\langle\left\langle \int \frac{1}{2}\epsilon_0 \hat{\vec{E}}(\vec{r})\hat{\vec{E}}(\vec{r}) + \frac{1}{2}\mu_0 \hat{\vec{H}}(\vec{r})\hat{\vec{H}}(\vec{r})\, d^3r \right\rangle\right\rangle$$

$$= \sum_{k\sigma} \hbar\omega_k\left(\langle\langle \hat{n}_{k\sigma}\rangle\rangle + \frac{1}{2}\right) = \sum_{k\sigma} \hbar\omega_k\left(\frac{1}{e^{\frac{\hbar\omega_k}{k_BT}}-1} + \frac{1}{2}\right)$$

(You can make a formula sheet for the exam)

$$\frac{\langle\langle \hat{H}\rangle\rangle}{L^3} \longrightarrow \int_0^\infty \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\hbar\omega_k/k_BT}-1}\,d\nu + \frac{Z.P.}{L^3}$$

$$u(\nu,T) = \frac{\text{energy}}{\text{vol}\cdot\text{freq}} = \frac{8\pi\nu^2}{c^3}\frac{h\nu}{e^{\hbar\omega/k_BT}-1} \longleftarrow \text{planck (1900)}$$

$$\psi = \sum_j a_j \phi_j e^{-iE_jt/\hbar}$$

$$\langle \hat{Q} \rangle = \langle \psi | \hat{Q} | \psi \rangle = \sum_j \sum_{j'} a_{j'}^+ a_j \langle \phi_{j'} | \hat{Q} | \phi_j \rangle\, e^{-[i(E_j - E_{j'})t]/\hbar}$$

$$\langle\langle \hat{Q} \rangle\rangle = \left\langle \sum_{j'}\sum_{j} a_{j'}^+ a_j \langle \phi_{j'} | \hat{Q} | \phi_j \rangle e^{i[(E_{j'}-E_j)/\hbar]t} \right\rangle_{th}\ (?)$$

$$= \sum_{i}\sum_{j} \langle a_{j'}^* a_j \rangle_{th} \langle \phi_{j'} | \hat{Q} | \phi_j \rangle\, e^{-\frac{i(E_j - E_{j'})}{\hbar}t}$$

$$\langle a_j^* a_j \rangle_{th} = \langle |a_j|^2 \rangle_{th} = P_j(T) \qquad \sqrt{\langle |a_j| \rangle_{th}^2} = \sqrt{P_j(T)}$$

$$\langle a_j^+ a_{j'} \rangle_{th} = \left\langle |a_j| e^{-i\theta_j} |a_{j'}| e^{i\theta_j} \right\rangle_{th} \sim \sqrt{P_j(t)}\,\sqrt{P_{j'}(t)} \left\langle e^{i(\theta_{j'} - \theta_j)} \right\rangle_{th}$$

$j' \neq j$

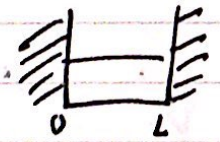
go from statistical mechanics to phase of formulation ...?

$$\frac{\int_0^{2\pi} d\theta_{j'} \int_0^{2\pi} d\theta_j \left\{ e^{i(\theta_{j'} - \theta_j)} \right\}}{\int_0^{\pi} d\theta_j \int_0^{2\pi} d\theta_{j'} \{1\}} = 0$$

$$\langle\langle \hat{Q} \rangle\rangle = \sum_j P_j(T) \langle \phi_j | \hat{Q} | \phi_j \rangle$$

- this will help with a part problem
- crucial for dielectric constant

Square Well

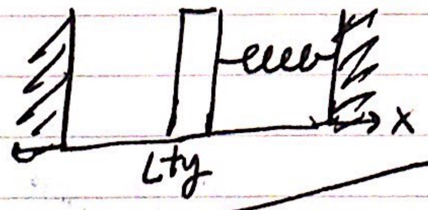


$$E_n = \frac{\hbar^2 \pi^2}{2mL^2} \qquad P_n(T) = \frac{e^{-E_n/k_BT}}{\sum_{n'} e^{-E_{n'}/k_BT}} = \frac{e^{-\frac{\hbar^2\pi^2}{2mL^2 k_BT} n^2}}{\sum_{n'} e^{-\frac{\hbar^2\pi^2}{2mL^2 k_BT} (n')^2}}$$

$$\langle\langle \hat{H} \rangle\rangle = \left\langle\left\langle \frac{\hat{p}^2}{2m} + V_{sq}(x) \right\rangle\right\rangle$$

$$= \frac{\sum_n E_n e^{-E_n/k_BT}}{\sum_n e^{-E_n/k_BT}}$$

Exam question ☆



$$\hat{H} = \frac{\hat{P}_x^2}{2m} + V_{sq}(0, L+y) + \frac{\hat{P}_y^2}{2m} + \frac{1}{2} M \omega_0^2 y^2$$

$$E_{n_x n_y} \approx \hbar\omega_0 \left(n_y + \frac{1}{2}\right) + \frac{\hbar^2 \pi^2}{2mL^2} n_x^2 - \frac{\left(\hbar^2\pi^2/(2mL^2)\, n_x^2\right)^2}{2M\omega_0^2}$$

$$\langle\langle \hat{H} \rangle\rangle = ?$$

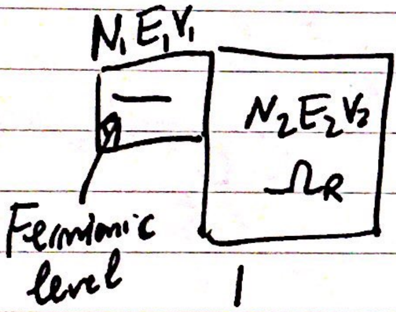
$$= \frac{\sum_{n_x} \sum_{n_y} E_{n_x n_y} e^{-\frac{E_{n_x n_y}}{k_BT}}}{\sum_{n_x} \sum_{n_y} e^{-\frac{E_{n_x n_y}}{k_BT}}}$$

$$E_{n_x n_y} = E_{n_x} + E_{n_y} \qquad \langle\langle \hat{H} \rangle\rangle = \frac{\sum_{n_x}\sum_{n_y}\left(E_{n_x}+E_{n_y}\right) e^{-\frac{E_{n_x}}{k_BT}} e^{-\frac{E_{n_y}}{k_BT}}}{}$$

$$= \left[\sum_{n_x}\sum_{n_y} E_{n_x} e^{-\frac{E_{n_x}}{k_BT}} e^{-\frac{E_{n_y}}{k_BT}}\right] / (\ldots) \quad \sum_{n_x}\sum_{n_y} e^{-\frac{E_{n_x}}{k_BT}} e^{-\frac{E_{n_y}}{k_BT}}$$

$$+ \frac{\sum_{n_x n_y} E_{n_y} e^{-\frac{E_{n_x}}{k_BT}} e^{-\frac{E_{n_y}}{k_BT}}}{\sum_{n_x}\sum_{n_y} e^{-\frac{E_{n_x}}{k_BT}} e^{-\frac{E_{n_y}}{k_BT}}}$$

$$= \frac{\sum_{n_x} E_{n_x} e^{-E_{n_x}/k_BT}}{\sum_{n_x} e^{-E_{n_x}/k_BT}} + \frac{\sum_{n_y} E_{n_y} e^{-\frac{E_{n_y}}{k_BT}}}{\sum_{n_y} e^{-E_{n_y}/k_BT}}$$



Configurations		N_1	E_1	V_1	Ω_1
no particle	—	0	0	V_0	1
1 particle	x	1	ϵ	V_0	1

$$\frac{1}{1+e^{-(\epsilon_0-\mu)/k_BT}} \qquad P_0(T) = \frac{1\cdot e^{-(0-\mu 0+pV_0)/k_BT}}{1\cdot e^{-(0-\mu 0+pV_0)/k_BT} + 1\cdot e^{-(\epsilon-\mu\cdot 1+pV_0)/k_BT}}$$

for $$\frac{e^{-(\epsilon-\mu)/k_BT}}{1+e^{-(\epsilon-\mu)/k_BT}} = P_1(T) = \frac{1\cdot e^{-(\epsilon-\mu\cdot 1+pV_0)/k_BT}}{\text{same}}$$

$$= \frac{1}{e^{(\epsilon-\mu)/k_BT}+1} \Bigg\} \text{Fermi-Dirac}$$

My interests:
device physics ?
less you invent things
probably need EE ...

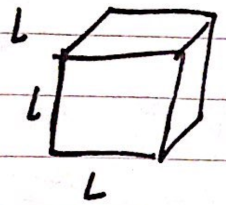
$48+6=$
54

$48+9$
57

$$\langle n \rangle_{th} = \sum_n^1 n P_n(T) = 0 p_0(t) + 1 \cdot P_1(T) = P_1(T) = \frac{1}{e^{\frac{\epsilon-\mu}{k_B T}}+1}$$

→ Application:

Example: electrons in a metal



$$n_e = \frac{\#\text{electrons}}{\text{volume}} = \frac{1}{L^3}\sum_j \langle n_j \rangle_{th}$$

spin degeneracy

$$g_s \frac{\pi}{4}\left(\frac{2mL^2}{\hbar^2\pi^2}\right)\epsilon^{1/2}$$

$$\rightarrow \frac{1}{L^3}\int_{-\infty}^{\infty} g(\epsilon)\frac{1}{e^{\frac{\epsilon-\mu}{k_B T}}+1}\,d\epsilon$$

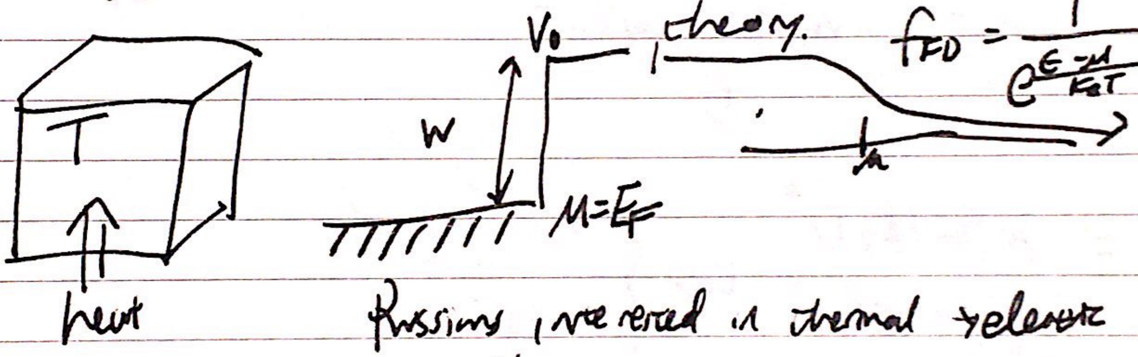
$\mu = E_F$ for electrons

have done a generalization

⇒ 1st angular relate to electron spectography (STM?)

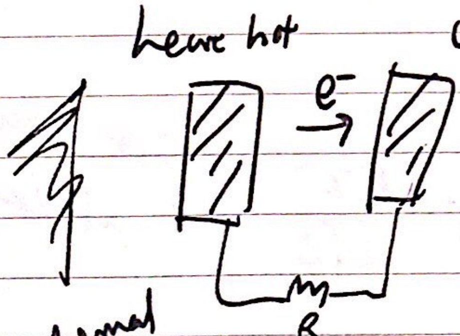
⇒ can sample electron distribution of crystal.

1st band observed ⇒ did not agree too much w/ theory.



$$f_{FD} = \frac{1}{e^{\frac{\epsilon-\mu}{k_B T}}+1}$$

Russians (were read in thermal +electric

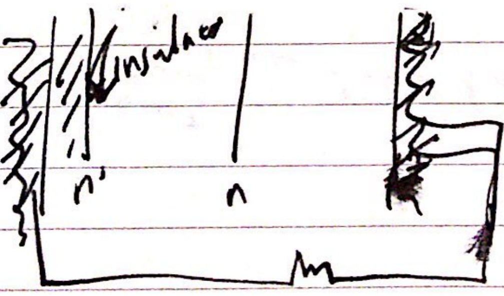


Idea: # e^- come out depends on work function.

oxides w/ low work functions ~.8eV

super thermal electric

phonontics company



Issue: electrons able to be ballistic

got 40% carnot

kinda worked

Thermal → electric conversion system.

$$J_z = \frac{1}{L^3}\sum_j q^{-|e|} (V_z)_j \frac{1}{e^{\frac{E_j-\mu}{k_BT}}+1} = \frac{1}{L^3}\sum_{\vec{k}\sigma} q V_z(\vec{k}) \frac{1}{e^{\frac{E(\vec{k})-\mu}{k_BT}}+1}$$

$\frac{1}{2} m v_z^2 > V_0$ $\qquad$ $\frac{1}{2} m v_z^2 > V_0$

$$\to g_s \int \frac{d^3\vec{k}}{(2\pi)^3} \frac{q\hbar k_z}{m} \frac{1}{e^{\frac{E(k)-\mu}{k_BT}}+1} = \frac{g_s q\hbar}{m}\int_{-\infty}^{\infty}\frac{dk_x}{2\pi}\int_{-\infty}^{\infty}\frac{dk_y}{2\pi}\int^{\infty}\frac{dk_z}{2\pi}$$

$$\frac{(\hbar k_z)^2}{2m} > V_0$$

$$k_z e^{-\frac{\hbar^2 k_x^2}{2mk_BT}} e^{-\frac{\hbar^2 k_y^2}{2mk_BT}}$$

$$e^{-\frac{\hbar^2 k_z^2}{2mk_BT}}$$

Simple exp expression

$$= \frac{g_s q\hbar}{m}\left[\int_{-\infty}^{\infty}\frac{dk}{2\pi} e^{-\frac{\hbar^2 k_x^2}{2mk_BT}}\right]\left[\text{same to } k_y\right]$$

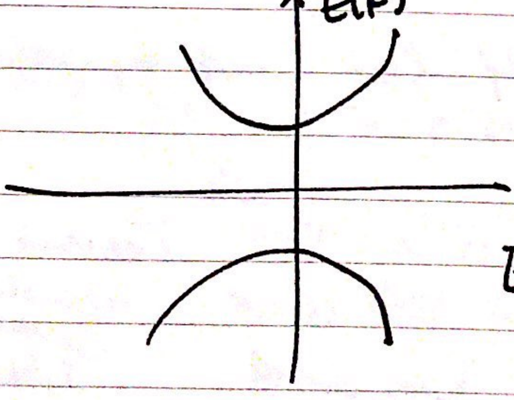
$$\left[\int_{k_m}^{\infty}\frac{dk_z}{2m} k_z e^{-\frac{\hbar^2 k_z^2}{2m}}\right]$$

$$\frac{1}{2\pi}\sqrt{\frac{2\pi m k_BT}{\hbar^2}} \to \text{"}$$

$$\frac{1}{2\pi}\left(\frac{mk_BT}{\hbar^2}\right) e^{-V_0/k_BT}\, e^{\mu/k_BT}$$

$$J_z = \frac{g_s q m (k_BT)^2}{(2\pi)^2\hbar^3}\exp\left\{\frac{-(\overbrace{V_0-\mu}^{W})}{k_BT}\right\}$$

Left: semiconductor models.



$$E_c(k) = E_c + \frac{\hbar^2k^2}{2m_e^*}$$

pset – doping w/ diamond

$$E_v(k) = E_v - \frac{\hbar^2k^2}{2m_h^*}$$