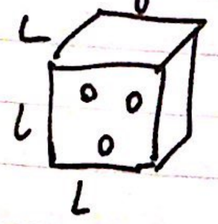
~~old pass of catch it.~~ Lecture 31

From last time:

Ideal gas



$$E = \frac{\hbar^2\pi^2}{2mL^2}\left((n_x^2)_1 + (n_y^2)_1 + (n_z^2)_1 + (n_y^2)_2 + \dots \right.$$

$$= \frac{\hbar^2\pi^2}{2mL^2} n^2 \qquad n^2 = (n_x^2)_1 + (n_y^2)_1 + (n_z^2)_1$$

$$N(n) = \frac{1}{2^{3N}} \frac{\pi^{3N/2}}{\left(\frac{3N}{2}\right)!} n^{3N}$$

$$N(E) = \frac{\frac{1}{2^{3N}} \frac{\pi^{3N/2}}{\left(\frac{3N}{2}\right)!} \left(\frac{2mL^2}{\hbar^2\pi^2} E \right)^{3N/2}}{N!} \quad \text{indistinguishable}$$

$$g(E) = \frac{\partial}{\partial E} N(E) \qquad \Omega(E) \approx g(E)\,\Delta E$$

$$S(N,E,V) = k_B \left\{ \frac{3N}{2}\ln\pi - 3N\ln 2 - \ln\left(\left(\frac{3N}{2}\right)!\right) - \ln N! + \frac{3N}{2}\ln\left|\frac{2m}{\hbar^2\pi^2}\right| + N\ln V + \ln\frac{3N}{2} + \left(\frac{3N}{2} - 1\right)\ln E + \ln\Delta E \right\}$$

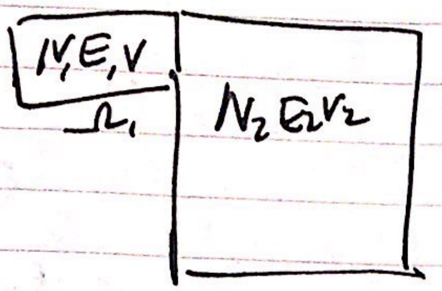
$$\rightarrow N k_B \left\{ \frac{3}{2}\ln\frac{E}{N} - \ln\frac{N}{V} + \frac{3}{2}\ln\left(\frac{m}{3\pi\hbar^2}\right) + \frac{5}{2} \right\}$$ Sakur-Tetrode entropy

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{NV} \Rightarrow E = \frac{3}{2} N k_B T$$

$$\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_{EN} \Rightarrow PV = N k_B T$$

$$-\frac{\mu}{T} = \left(\frac{\partial S}{\partial N}\right)_{EV} \Rightarrow -\frac{\mu}{T} = \frac{3}{2}k_B \ln\left(\frac{E}{N}\right) - k_B\ln\left(\frac{N}{V}\right) + \frac{3}{2}k_B \ln\frac{m}{3\pi\hbar^2}$$

Reservoir + Microsystem



$\Omega_2 = \Omega_R$

$$\Omega(NEV) = \sum_{N_1 E_1 V_1} \Omega_1(N_1 E_1 V_1)\, \Omega_R(N-N_1, E-E_1, V-V_1)$$

$$P(N_1 E_1 V_1) = \frac{\Omega_1(N_1 E_1 V_1)\, \Omega_2(N-N_1, E-E_1, V-V_1)}{\sum_{NEV} \Omega_1(N_1' E_1' V_1')\, \Omega_R(N-N_1', E-E_1', V-V_1')}$$

$\Omega(NEV)$

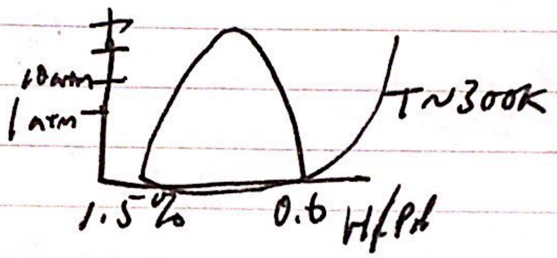
$S = k_B \ln \Omega$

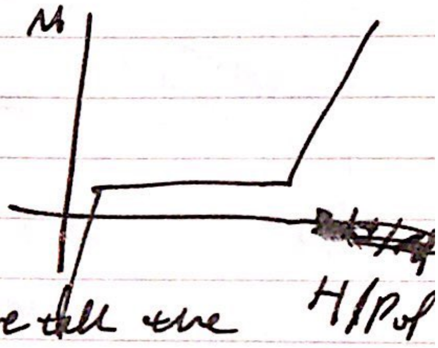
$\Omega_R = e^{S_R/k_B}$

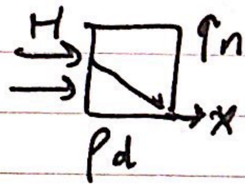
$$S_R(N-N_1, E-E_1, V-V_1) = S_R(NEV) - N_1\left(\frac{\partial S_R}{\partial N}\right)_{EV} - E_1\left(\frac{\partial S_R}{\partial E}\right)_{NV} - V_1\left(\frac{\partial S_R}{\partial V}\right)_{EN}$$

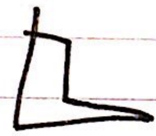
$\vec{J}_0 = -D\nabla_n$

$\vec{J}_D = -B\nabla_\mu$









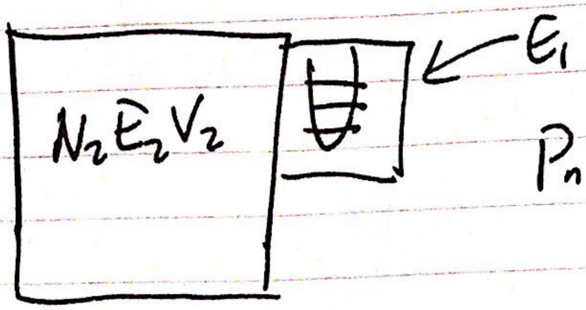
How do we tell the difference?

$$p(N_1 E_1 V_1) = \frac{\Omega_1(N_1 E_1 V_1)\, e^{S_R(NEV)/k_B} e^{(\,)}}{\sum_{N_1 E_1 V_1} \Omega_1(N_1' E_1' V_1')\, e^{S_R(NEV)/k_B} e^{(\,)}}$$

$= \Omega_2(NEV)$

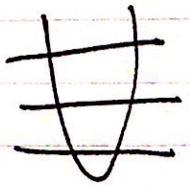
$$\boxed{p(N,E,V) = \frac{\Omega_1(N_1 E_1 V_1)\, e^{-E_1/k_BT} e^{N_1\mu/k_BT} e^{-pV_1/k_BT}}{\sum_{N_1 E_1 V_1} \Omega_1(N_1' E_1' V_1')\, e^{-E_1'/k_BT} e^{\mu N_1/k_BT} e^{-pV_1'/k_BT}}}$$

$\Omega_2(NEV)$



$$P_n(T) = \frac{1 \cdot e^{-E_n/k_BT}\, e^0 \cdot e^0}{\sum_{n'} e^{-E_{n'}/k_BT}} = \frac{e^{-E_n/k_BT}}{\sum_{n'} e^{-E_{n'}/k_BT}}$$

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega_0^2 x^2$$



$$E_n = \hbar\omega_0 (n + \tfrac{1}{2})$$

$$P_n(T) = \frac{e^{-\frac{\hbar\omega_0(n+1/2)}{k_BT}}}{\sum_{n'} e^{-\frac{\hbar\omega_0(n'+\frac{1}{2})}{k_BT}}} = \frac{e^{-\frac{\hbar\omega_0}{k_BT} n}}{\sum_{n'} e^{-\frac{\hbar\omega_0}{k_BT} n'}}$$

$$\langle\langle \hat{H} \rangle\rangle = \langle\langle \frac{\hat{p}^2}{2m} + \frac{1}{2} m\omega_0^2 x^2 \rangle\rangle$$

$$= \langle\langle \hbar\omega_0 (\hat{a}^\dagger \hat{a} + \tfrac{1}{2}) \rangle\rangle \qquad \rightarrow \frac{1}{1 - e^{-\hbar\omega_0/(k_BT)}}$$

quantum mechanical avg

thermal avg

$$= \sum_n E_n p_n(T) = \frac{\sum_n E_n e^{-E_n/k_BT}}{\sum_n e^{-E_n/k_BT}}$$

$$= \frac{\sum_n \hbar\omega_0 (n+\frac{1}{2}) e^{-\hbar\omega_0/(k_BT)\,[n+\frac{1}{2}]}}{\sum_n e^{-\frac{\hbar\omega_0}{k_BT}(n+\frac{1}{2})}}$$

$$= \frac{1}{2}\hbar\omega_0 + \hbar\omega_0 \frac{\sum_n n\, e^{-\frac{\hbar\omega_0}{k_BT} n}}{\sum_n e^{-\frac{\hbar\omega_0}{k_BT} n}}$$

$$F(\beta) = \sum_{n=0}^{\infty} e^{-\beta n} = 1 + e^{-\beta} + (e^{-\beta})^2 + \ldots = \frac{1}{1 - e^{-\beta}} \qquad |e^{-\beta}| < 1$$

$$\beta = \frac{\hbar\omega_0}{k_BT} \qquad \sum_{n=0}^{\infty} n\, e^{-\beta n} = -\frac{d}{d\beta} F(\beta) = -\frac{d}{d\beta}\frac{1}{1 - e^{-\beta}} = \frac{e^{-\beta}}{(1 - e^{-\beta})^2}$$

$$\langle\langle \hat{H} \rangle\rangle = \frac{1}{2}\hbar\omega_0 + \hbar\omega_0 \frac{\frac{e^{-\hbar\omega_0/k_BT}}{(1 - e^{-(\,)})^2}}{1/(1 - e^{-(\,)})} = \frac{e^{-\hbar\omega_0/k_BT}}{1 - e^{-\hbar\omega_0/k_BT}}$$

$+ZP$

$$= \frac{\hbar\omega_0}{e^{\hbar\omega_0/k_BT}-1} + \frac{1}{2}\hbar\omega_0$$

$$\langle\langle \hat{H} \rangle\rangle = \langle\langle \int \frac{1}{2}\epsilon_0 \hat{\vec{E}}\cdot\hat{\vec{E}} + \frac{1}{2}\mu_0 \hat{\vec{H}}\cdot\hat{\vec{H}}\, d^3r \rangle\rangle$$

$$= \langle\langle \sum_{k\sigma} \hbar\omega_k (\hat{n}_{k\sigma} + \tfrac{1}{2}) \rangle\rangle$$

$$= \sum_{k\sigma} \hbar\omega_k \left(\langle\langle \hat{n}_{k\sigma} \rangle\rangle + \frac{1}{2} \right)$$

$$\langle\langle \hat{H}_{k\sigma} \rangle\rangle = \sum_{k\sigma} \left(\frac{\hbar\omega_k}{e^{\frac{\hbar\omega_k}{k_BT}}-1} + \frac{\hbar\omega_k}{2} \right)$$

$$\langle\langle \hat{H} \rangle\rangle = \sum_{k\sigma} \frac{\hbar\omega_k}{2} + \int_{-\infty}^{\infty} \rho(\epsilon) \frac{\epsilon}{e^{\epsilon/kT}-1}\, d\epsilon$$

$$\rho(\epsilon) = \begin{cases} 0 & \epsilon < 0 \\ \dfrac{L^3\epsilon^2}{\pi^2\hbar^3c^3} & \epsilon > 0 \end{cases} \qquad \int_0^{\infty} \frac{L^3\epsilon^3}{\pi^2\hbar^3c^3} \frac{1}{e^{\epsilon/k_BT}-1}\, d\epsilon$$

$$\epsilon = h\nu$$

$$\frac{\langle\langle \hat{H} \rangle\rangle - Z_1 P_1}{L^3} = \int_0^{\infty} \frac{(h\nu)^3}{\pi^2 \left(\frac{h}{2\pi}\right)^3 c^3} \frac{1}{e^{h\nu/k_BT}-1}\, h\, d\nu$$

$$= \int_0^{\infty} \frac{8\pi\nu^3}{c^3} \frac{h\nu}{e^{h\nu/k_BT}-1}\, d\nu$$

$$u(\nu, T) = \frac{8\pi\nu^2}{c^3} \frac{h\nu}{e^{h\nu/k_BT}-1}$$

planck's formula