11/18 Lecture 30

Golden Rule $\gamma = \frac{2\pi}{\hbar} |\langle f | \hat{H}_{int} | i \rangle|^2 \rho$

Radioactive decay $\gamma = \frac{2\pi}{\hbar} \overline{|\langle \Psi_{k\sigma} | \hat{H}_{int} | \Psi_0 \rangle|^2} \rho$

averaged over angle & polarization

$\langle \Psi_{k\sigma} | \hat{H}_{int} | \Psi_0 \rangle = \langle \Psi_{k\sigma} | -\frac{q}{m} \hat{A}(r) \hat{p} | \psi_0 \rangle$

$= \frac{-iq}{m} \sqrt{\frac{\hbar}{2\epsilon_0 \omega_k L^3}} \hat{\imath}_\sigma \langle \phi_f | \hat{p} | \phi_i \rangle$

$\gamma = \frac{2\pi}{\hbar} \overline{\left| \frac{-iq}{m} \sqrt{\frac{\hbar}{2\epsilon_0 \omega_k L^3}} \hat{\imath}_\sigma \langle \phi_f | \hat{p} | \phi_i \rangle \right|^2}$ $\quad \frac{L^3 (\hbar \omega_k)^2}{\pi^2 \hbar^3 c^3}$

$\hbar \omega_k = H_{22} - H_{11}$

$= 4 \frac{q^2}{4\pi\epsilon_0} \frac{\omega}{\hbar m^2 c^3} \overline{|\hat{\imath}_\sigma \langle \phi_f | \hat{p} | \phi_i \rangle|^2}$

$\overline{} = \frac{1}{3} |\langle \phi_f | \hat{p} | \phi_i \rangle|^2$

$\gamma = \frac{4}{3} \frac{q^2}{4\pi\epsilon_0} \frac{\omega}{\hbar m^2 c^3} |\langle \phi_f | \hat{p} | \phi_i \rangle|^2$

Statistical Mechanics

$S = k_B \ln \Omega \quad \Omega = \#$ accessible microstates

$k_B = 1.381 \times 10^{-23}$ J/K

$= 8.617 \times 10^{-5}$ eV/K

$N_1\ E_1\ V_1$	$N_2\ E_2\ V_2$

$\Omega(N, E, V) = \sum_N \sum_E \sum_V \Omega_1(N_1 E_1 V_1) \Omega_2(N_2 E_2 V_2)$

$N_1 + N_2 = N, V_1 + V_2 = V, E_1 + E_2 = E$

Fundamental Postulate in thermal equilibrium are equally probable

$$\frac{\partial}{\partial E_1}(\Omega_1\Omega_2)=0 \Rightarrow \left(\frac{\partial S_1}{\partial E_1}\right)_{N_1V_1} = \left(\frac{\partial S_2}{\partial E_2}\right)_{N_2V_2}$$

$$\Rightarrow \frac{1}{T_1} = \frac{1}{T_2}$$

$$\frac{\partial}{\partial V_1}(\Omega_1\Omega_2)=0 \qquad \left(\frac{\partial S_1}{\partial V_1}\right)_{E_1N_1} = \left(\frac{\partial S_2}{\partial V_2}\right)_{E_2V_2}$$

$$\Rightarrow \frac{P_1}{T_1} = \frac{P_2}{T_1}$$

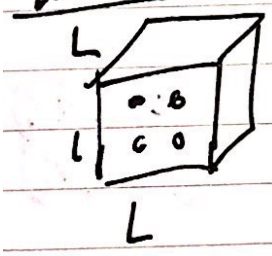
$$\frac{\partial}{\partial N_1}(\Omega_1\Omega_2)=0 \Rightarrow \left(\frac{\partial S_1}{\partial N_1}\right)_{E_1N_1} = \left(\frac{\partial S_2}{\partial N_2}\right)_{E_2V_2}$$

$$\Rightarrow -\frac{\mu_1}{T_1} = -\frac{\mu_2}{T_2}$$

$$S=S(N,E,V) \Rightarrow dS = \left(\frac{\partial S}{\partial E}\right)_{NV} + \left(\frac{\partial S}{\partial N}\right)_{EV} dN$$

$$+\left(\frac{\partial S}{\partial V}\right)_{NE} = \frac{dE}{T} - \frac{\mu}{T}dN + \frac{P}{T}dV$$

<u>Ideal Gas</u> Non-interacting



$$E = \frac{\hbar^2\pi^2}{2mL^2}\Big((n_x)_1^2 + (n_y)_1^2 + (n_z)_1^2$$
$$+ (n_x)_2^2 + (n_y)_2^2 + (n_z)_2^2$$
$$\vdots \Big)$$

$$\Omega(N,E,V) \approx \Delta E\, g(E) \leftarrow \text{density of states}$$

$$N = \frac{1}{8}\left(\frac{4\pi}{3}n^2\right)$$ 1 particle, 3D box

$$N(n) = \frac{1}{2^{3N}}\left(\frac{\pi^{3N/2}}{\left(\frac{3N}{2}\right)!}\right)n^{3N}$$ N particle 3N-D box (N entry)

$$N(E) = \left[\frac{1}{2^{3N}}(\;u\;)\left(\frac{2mL^2E}{\hbar^2\pi^2}\right)^{3N/2}\right]\Big/ N!$$ p_1 p_2

$$g(E) = \frac{\partial}{\partial E} N(E) = \frac{1}{2^{3N}} \frac{\pi^{\frac{3N}{2}}}{\left(\frac{3N}{2}\right)!} \frac{1}{N!} \left(\frac{2mL^3}{\hbar^2 \pi^2}\right)^{3N/2} \frac{3N}{2} E^{3N/2-1}$$

$$\Omega = g \Delta E$$

$$S = k_B \ln \Omega = k_B \left\{ \frac{3N}{2} \ln \pi - 3N \ln 2 - \ln\left(\frac{3N}{2}\right)! - \ln N! + \frac{3N}{2} \ln\left[\frac{2m}{\hbar \pi^2}\right] + N \ln V + \ln \frac{3N}{2} + \left(\frac{3N}{2} - 1\right) \ln E + \ln \Delta E \right\}$$

$$S(NEV) = N k_B \left\{ \frac{3}{2} \ln \frac{E}{N} - \ln \frac{N}{V} + \frac{3}{2} \ln\left(\frac{m}{3\pi\hbar^2}\right) + \frac{5}{2} \right\}$$

Larger N limit

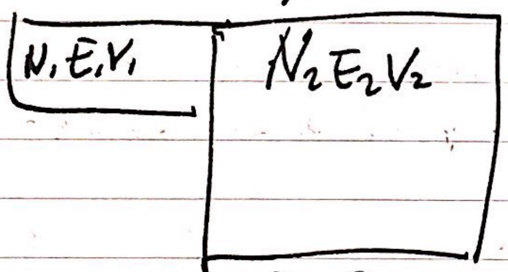
Sackur – Tetrode entropy 1912

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{NV} \Rightarrow E = \frac{3}{2} N k_B T$$

$$\frac{P}{T} = \left(\frac{\partial S}{\partial V}\right)_{NE} \Rightarrow PV = N k_B T$$

$$-\frac{\mu}{T} = \left(\frac{\partial S}{\partial N}\right)_{EV} = \frac{3}{2} k_B \ln \frac{E}{N} - k_B \ln \frac{N}{V} + \frac{3}{2} k_B \ln \frac{m}{3\pi\hbar^2}$$

Two systems in equilibrium



$\Omega_2 = \Omega_R$

$N_1 + N_2 = N$

$E_1 + E_2 = E$

$V_1 + V_2 = V$

$$\Omega(NEV) = \sum_{N_1} \sum_{E_1} \sum_{V_1} \Omega_1(N_1 E_1 V_1)\, \Omega_R(N_2 E_2 V_2)$$

$$P = \frac{1}{\Omega}$$

$$p = \frac{\Omega_1(N_1 E_1 V_1)\, \Omega_R(N_2 E_2 V_2)}{\sum_{N_1} \sum_{E_1} \sum_{V_1} \Omega_1(N_1, E_1, V_1)\, \Omega_R(N_2 E_2 V_2)}$$

$$\Omega_R(NEV) = e^{S(NEV)/k_B}$$

Taylor series expansion... for linearization.