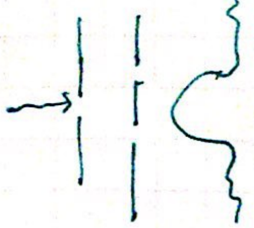
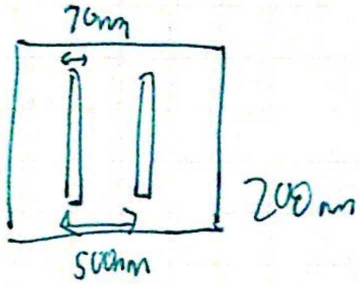
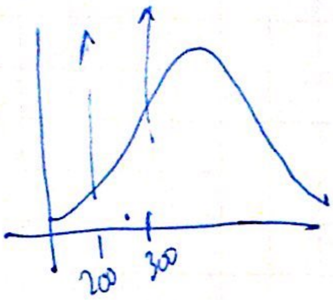
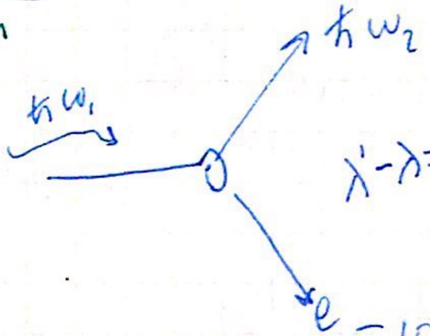
Lecture 3





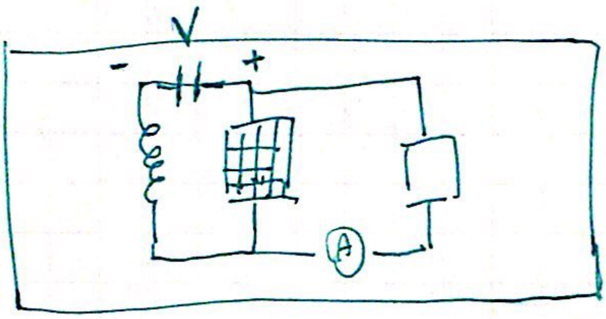
more into light than just it being a wave.

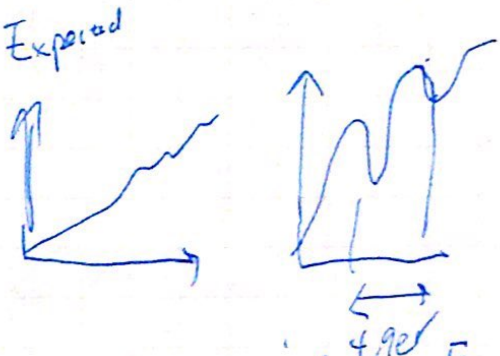


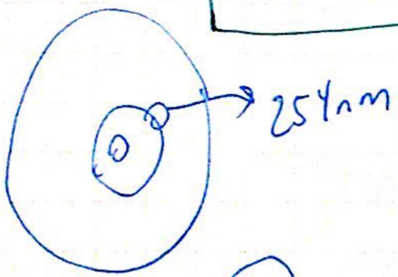


$$\lambda' - \lambda = \frac{\hbar}{m_e c}(1-\cos\theta)$$

$$= 10^{-12}(1-\cos\theta)$$







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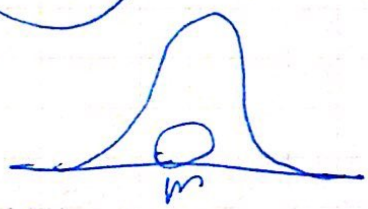
particle shows some wave properties

spatial e^{ikx} E time evolution $e^{-i\omega t}$

$\hbar\omega = E$ $\Psi(x_0, t) =$

$\hbar k = p$ $\boxed{\omega = \frac{\hbar k^2}{2m}}$

$\Rightarrow \hbar\omega = \frac{p^2}{2m} = \frac{\hbar^2 k^2}{2m}$



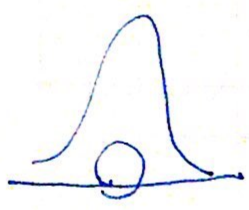
$$\Psi(x,t) = \int A(k) e^{ikx} e^{-\frac{i\hbar k^2}{2m}t} \frac{dk}{2\pi}$$

$$|\Psi(x,t)|^2$$

interpret wave as probability, can interpret double slit experiment

$$\frac{\partial^2 \Psi(x,t)}{\partial x^2} = -k^2 \Psi(x,t)$$

$$\frac{\partial \Psi(x,t)}{\partial t} = \frac{-i\hbar k^2}{2m}\Psi(x,t) \Rightarrow \frac{-\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + \frac{i\hbar\,\partial\Psi}{\partial t} = 0$$



$t=0$

$$\psi(x,0) = e^{-\frac{\pi}{2}x^2}$$

$$\psi(x,0) = \int A(k) e^{ikx} \frac{dk}{2\pi}$$

this is the fourier transform

we should be able to extract $A(k)$...

$$A(k) = \int \psi(x,0) e^{-ikx} dx$$

$$P(x) = \int F(k) e^{ikx} \frac{dk}{2\pi} \qquad \psi = e^{-\alpha x^2}$$

$$F(k) = \int F(x) e^{-ikx} dx \qquad \psi' + 2\alpha \psi = 0$$

$$\frac{df}{dx} = ? \qquad ikF(k) \qquad ikA(k) + 2\alpha i \frac{dF}{dk} = 0 \Rightarrow \frac{dF}{dk} + \frac{k}{2\alpha} F = 0$$

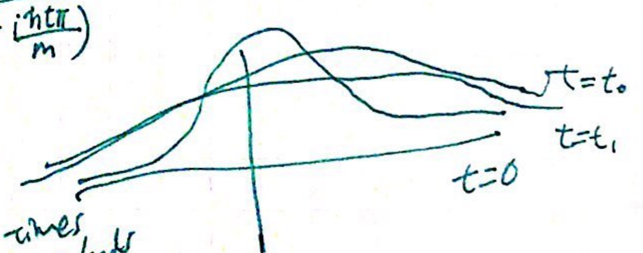
$$xf(x) \to \frac{i\,dF}{dk}$$

$$\Rightarrow \boxed{A(k) = \sqrt{2}\, e^{-\frac{1}{2\pi}k^2}}$$

k is spreading?
how to relate k to other parameters we're more familiar with

$$\psi(x,t) = \sqrt{2} \int e^{-\frac{1}{2\pi}k^2} e^{ikx} e^{-\frac{i\hbar k^2}{2m}t} \frac{dk}{2\pi}$$

$$\psi(x,t) = \frac{1}{\sqrt{1+\frac{i\hbar\pi t}{m}}} e^{\frac{-\pi x^2}{2\left(1+\frac{i\hbar t\pi}{m}\right)}}$$



model doesn't allow molecule to stay at rest

model has → uncertainties between certain variables.

At later times the gaussian spreads

spreading momentum
spread velocity

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 \qquad \langle x^2 \rangle_{t=0} = \int x^2 |\psi(x,t)|^2 dx = \int x^2 e^{-\pi x^2} dx = \frac{1}{2\pi}$$

Boltzmann integrals

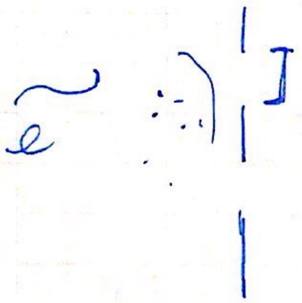
$\int e^{-\alpha x^2} = I$ $\qquad I^2 = \int e^{-\alpha(x^2+y^2)}\,dx\,dy$

$= \int_0^{2\pi} e^{-\alpha r^2}\, r\,dr\,d\varphi = \sqrt{\frac{\pi}{4}}$

$-\frac{d}{d\alpha} = \frac{1}{2}\frac{\sqrt{\pi}}{4\sqrt{\alpha}}$ $\qquad \langle k^2\rangle = \int k^2 \frac{|A(k)|^2}{2\pi} = \frac{\pi}{2}$

$\sigma_n \sigma_k = \frac{1}{2}$

hit with a very small wavelength to see where photon is $\Rightarrow$ high energy



$\langle x^2\rangle = \frac{1}{2\pi}\left(1+\frac{\hbar^2 t^2 \pi^2}{m^2}\right) \Rightarrow \langle x^2\rangle_t = \langle x^2\rangle_{t=0} + \langle v^2\rangle t^2$

$\langle k^2\rangle = \pi/2$ $\qquad p = \hbar k$

$= \hbar^2\langle p^2\rangle = \hbar^2 m^2\langle v^2\rangle = \frac{\pi}{2}$

$\langle v^2\rangle = \frac{\hbar^2 \pi}{2m^2}$

principle of least action

$i^i = e^{-\pi/2}$!

office hours at 4

MTF