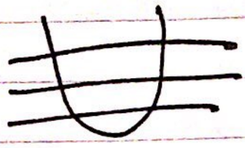
Lecture 29

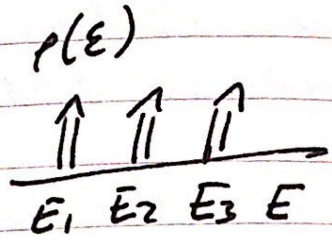
dipole transitions

From last time

Density of states



$$\rho(E) = \sum_j \delta(E - E_j)$$



$$\rightarrow \begin{cases} \frac{1}{\hbar\omega_0} & E>0 \\ 0 & E<0 \end{cases} \quad \text{for SHO}$$

$$\gamma = \frac{2\pi}{\hbar} |\langle \phi_f | \hat{V} | \phi_i \rangle|^2 \rho$$

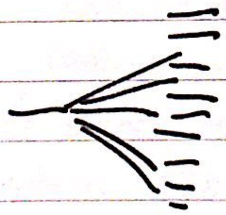
Radiation Field

$$\sum_K \sum_\sigma Q_\sigma(\vec{k}) \longrightarrow \sum_\sigma \int Q_\sigma(\vec{k}) \frac{d^3\vec{k}}{\left(\frac{2\pi}{L}\right)^3} = \sum_\sigma L^3 \int Q_\sigma(\vec{k}) \frac{d^3\vec{k}}{(2\pi)^3}$$

$$\vec{k} = \hat{i}_x \frac{2\pi n_x}{L} + \hat{i}_y \frac{2\pi n_y}{L} + \hat{i}_z \frac{2\pi n_z}{L}$$

$$\rho(\varepsilon) = \begin{cases} 0 & E<0 \\ \frac{L^3 E^2}{\pi^2 \hbar^3 c^3} \end{cases}$$

Radiative Decay



$$\Psi_{k\sigma} = \phi_f(\vec{r})\, \hat{a}^\dagger_{k\sigma} \Phi_0$$

$$\Psi_i = \phi_i(\vec{r})\, \Phi_0$$

$$\langle \Psi_{k\sigma} | \hat{H}_{int} | \Psi_i \rangle$$

$$= \left\langle \phi_f(\vec{r}) \hat{a}^\dagger_{k\sigma} \Phi_0 \right| \frac{-q}{m} \sum_{k\sigma} \hat{i}_\sigma \sqrt{\frac{\hbar}{2\varepsilon_0 \omega_k L^3}} \qquad = \left\langle \Psi_{k\sigma} \right| \frac{-q}{m} \vec{A}(\vec{r})\, \hat{p} \left| \Psi_i \right\rangle$$

$$\left(\hat{a}_{k\sigma} e^{ik\cdot r} - \hat{a}^\dagger_{k\sigma} e^{ik\cdot r} \cdot \hat{p} \,\middle|\, \phi_i(\vec{r}) \Phi_0 \right\rangle = \frac{-iq}{m} \sqrt{\frac{\hbar}{2\varepsilon_0 \omega_k L^3}} \langle \phi_f(\vec{r}) | e^{-i\vec{k}\cdot\vec{r}}\, \hat{i}_\sigma \hat{p} | \phi_i(\vec{r}) \rangle$$

$$e^{-i\vec{k}\cdot\vec{r}} \rightarrow 1 \quad \text{often}$$

$$\langle \Psi_{k\sigma} | \hat{H}_{int} | \Psi_i \rangle \rightarrow -i \frac{q}{m} \sqrt{\frac{\hbar}{2\varepsilon_0 \omega_k L^3}} \langle \phi_f | \hat{p} | \phi_i \rangle \hat{i}_\sigma = V^*_{k\sigma}$$

$$\gamma \overset{?}{=} \frac{2\pi}{\hbar} |\langle \Psi_{k\sigma} | \hat{H}_{int} | \Psi_i \rangle|^2 \rho(E) \Big|_{E=\hbar\omega}$$

$$i\hbar \frac{d}{dt} C_0(t) = E_0 C_0(t) + \sum_{k\sigma} V_{k\sigma} C_{k\sigma}(t)$$

$$i\hbar \frac{d}{dt} C_{k\sigma}(t) = E_{\vec{k}} C_{k\sigma}(t) + V^*_{k\sigma} C_0(t)$$

$$C_{k\sigma}(t) = \frac{1}{i\hbar} V^*_{k\sigma} \int_0^t e^{-\frac{iE}{\hbar}(t-t')} C_0(t')\,dt'$$

$$i\hbar \frac{d}{dt} C_0 = E_0 C_0 + \frac{1}{i\hbar} \sum_{k\sigma} |V_{k\sigma}|^2 \int_0^t e^{-\frac{iE_k(t-t')}{\hbar}} C_0(t')\,dt'$$

$$\rightarrow E_0 C_0 + \frac{1}{i\hbar} \sum_\sigma L^3 \int \frac{d\vec{k}^3}{(2\pi)^3} |V_\sigma(k)|^2 \int_0^t e^{-iE_k(t-t')} C_0(t')\,dt'$$

$$\frac{1}{i\hbar} \sum_\sigma \frac{L^3}{(2\pi)^3} \int_0^\infty k^2 dk \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi\, |V_\sigma(k)|^2 \int_0^t \cdots dt'$$

$$\epsilon = \hbar\omega_k = \hbar c k \quad k = \frac{\epsilon}{\hbar c}$$

$$\frac{1}{i\hbar} \frac{L^3}{(2\pi)^3} \sum_\sigma \int_0^\infty \left(\frac{\epsilon}{\hbar c}\right)^2 \frac{d\epsilon}{(\hbar c)} \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi\, |V_\sigma(\epsilon)|^2 \int_0^t (\quad)\,dt'$$

$$\rho(\epsilon) = \begin{cases} 0 & \epsilon < 0 \\ \frac{L^3 \epsilon^2}{\pi^2 4 (\hbar c)^3} & \epsilon > 0 \end{cases}$$

$$\rightarrow \frac{1}{i\hbar} \frac{1}{8\pi} \sum_\sigma \int_{-\infty}^\infty \rho(\epsilon)\, d\epsilon \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\theta\, |V_\sigma(\epsilon)| \int_0^t (\quad)\,dt'$$

$$= \frac{1}{i\hbar} \int_{-\infty}^\infty \rho(\epsilon)\, dE \sum_\sigma \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi\, |V_\sigma(\epsilon)|^2 \int_0^t (\ldots)\,dt'$$

$$\sum_\sigma \int_0^\pi \sin\theta\, d\theta \int_0^{2\pi} d\phi$$

$$E_0 C_0 + \frac{1}{i\hbar} \int_{-\infty}^\infty \rho(\epsilon)\, d\epsilon\, \overline{|V_\sigma(\epsilon)|^2} \int_0^t (\quad) C_0(t')\,dt' = i\hbar \frac{d}{dt} C_0$$

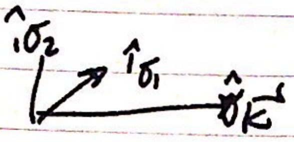
$|V|^2 \rho(\epsilon) \rightarrow$ constant $\qquad \frac{1}{i\hbar} \int_0^t f(t-t') C_0(t')\,dt'$

$f(\tau) = (\,)\,e^{-E\tau i/\hbar}$
$((\,) + (\,)\tau + (\,)\tau^3 + (\,)\tau^2 + \tau)^4)$?
Not sure what's going on

"Formula for dirac model"

$$|\langle\phi_f|\hat{\vec{p}}|\phi_i\rangle\cdot\hat{i}_\sigma|^2 \qquad |\vec{Q}|^2 = \vec{Q}^*\cdot\vec{Q} = |Q_x|^2+|Q_y|^2+|Q_z|^2$$

$$= |\hat{i}_x\cdot\vec{Q}|^2+|\hat{i}_y\cdot\vec{Q}|^2+|\hat{i}_z\cdot\vec{Q}|^2$$



orthogonal

$$|\vec{Q}|^2 = |\hat{i}_{\sigma_1}\cdot\vec{Q}|^2+|\hat{i}_{\sigma_2}\cdot\vec{Q}|^2+|\hat{i}_k\cdot\vec{Q}|^2$$

"Summing over polarizations"

$$\gamma = \frac{2\pi}{\hbar}\overline{|\langle f|\hat{H}_{int}|i\rangle|^2_P}$$

$$= \frac{2\pi}{\hbar}\frac{L^3\epsilon^2}{\pi^2\hbar^3c^3}\left|-i\frac{q^2}{m}\sqrt{\frac{\hbar}{2\epsilon\omega_k L^3}}\langle\phi_f|\vec{p}|\phi_i\rangle\right|^2\frac{1}{3} =$$

$$\boxed{\frac{4}{3}\frac{q^2}{4\pi\epsilon_0}\frac{\omega}{\hbar m^2c^3}|\langle\phi_f|\hat{\vec{p}}|\phi_i\rangle|^2}$$

end of this ~~sect~~ section:

- quantization of electric field leads to the ~~calcula~~ calculation of the decay rate.

Statistical Mechanics

Classical Thermodynamics

Empirical:

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{N,V}$$

Boltzmann: $S = k_B \ln \Omega$

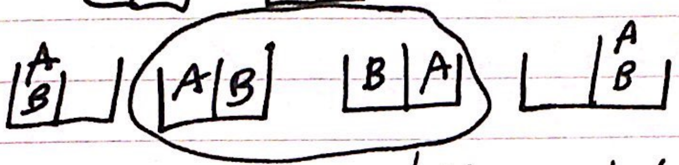
k_B: $1.381 \times 10^{-23} \frac{J}{K}$, $8.617 \times 10^{-5} \frac{eV}{K}$

Ω: # of accessible microstates

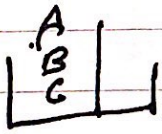
Socks

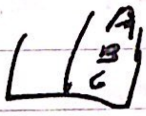
Put socks in two drawers


$\Rightarrow$ 2 ways to put socks


$\Rightarrow$ 4 ways to put socks

higher probability of having a configurable equal number in both.

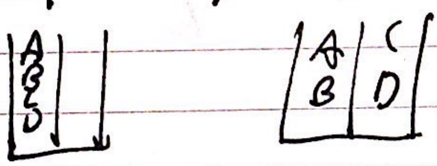

6 (...)



how many configurations do I have in the most probable configuration?

$N = 4$

1 4 6 4 1



$$\binom{N}{m} = \frac{N!}{m!(N-m)!}$$

Steve's theorem

$$\ln\left(\begin{array}{c}\text{\# of states of}\\ \text{most probable}\\ \text{configuration}\end{array}\right) \approx \ln\left(\begin{array}{c}\text{total \#}\\ \text{of states}\end{array}\right)$$

$\binom{N}{N/2}$ $\qquad$ 2^N

$N = 100$

$\ln \Omega = \ln 2^{100} = 100 \ln(2)$

$= 69.315$

$\ln \Omega_{\text{most probable}} = \ln \binom{100}{50} = 66.7779$

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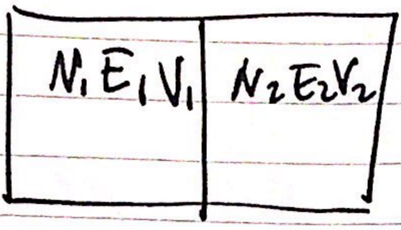


$N = N_1 + N_2$

$E = E_1 + E_2$

$V = V_1 + V_2$

$$\Omega(NEV) = \sum_{N_1E_1V_1} \Omega_1(N_1E_1V_1)\,\Omega_2(N_2E_2V_2)$$

$$\ln \Omega(NEV) \approx \ln \Omega(N_1E_1V_1)\Omega(N_2E_2V_2) \xrightarrow{\text{most probable}}$$

$$\frac{\partial}{\partial E_1}\left(\Omega_1(N_1E_1V_1)\,\Omega_2(N-N_1, E-E_1, V-V_1)\right)$$

$$= \left(\frac{\partial \Omega_1}{\partial E_1}\right)_{N_1V_1} \Omega_2(N_2E_2V_2) + \left(\frac{\partial \Omega_2}{\partial E_1}\right) \Omega_1(N_1E_1V_1) = 0$$

$$\frac{1}{\Omega_1}\left(\frac{\partial \Omega_1}{\partial E_1}\right)_{N_1V_1} = -\frac{1}{\Omega_2}\left(\frac{\partial \Omega_2}{\partial E_1}\right)_{N_1V_1} = \frac{1}{\Omega_2}\left(\frac{\partial \Omega_2}{\partial E_2}\right)_{N_2V_2}$$

$$\left(\frac{\partial \ln \Omega_1}{\partial E_1}\right)_{N_1V_1} = \left(\frac{\partial \ln \Omega_2}{\partial E_2}\right)_{N_2V_2}$$

punchline: multiply by Boltzmann constant

$$\left(\frac{\partial}{\partial E_1} k_B \ln \Omega_1\right)_{N_1V_1} = \left(\frac{\partial S_1}{\partial E_1}\right) = \left(\frac{\partial S_2}{\partial E_2}\right)_{N_2V_2}$$

$\frac{1}{T_1}$ $\frac{1}{T_2}$

$\Downarrow$ $T_1 = T_2$

$$\frac{1}{T_1} = \frac{1}{T_2}$$

In thermal equilibrium

⇒ ~~most probable~~ ~~all accessible~~ microstate

⇒ system ~~goes to most probable microstate~~.

⇒ all accessible microstates equally probable.

the one mostly likely is $T_1 = T_2$

Same thing with volume.

$$\left(\frac{\partial S_1}{\partial V_1}\right)_{N_1 E_1} = \left(\frac{\partial S_2}{\partial V_2}\right)_{N_2 E_2}$$

$$\to \frac{P_1}{T_1} = \frac{P_2}{T_2}$$

Number

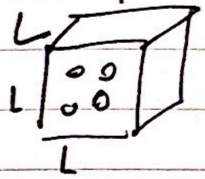
$$\left(\frac{\partial S_1}{\partial N_1}\right)_{E_1 V_1} = \left(\frac{\partial S_1}{\partial N_2}\right)_{E_2 N_2}$$

$$\downarrow \qquad\qquad \downarrow$$

$$-\frac{\mu_1}{T_1} = -\frac{\mu_2}{T_2}$$

$$\Rightarrow \mu_1 = \mu_2$$

Example: Non-interacting



If one particle, how many accessible microstates?

$$\Omega \approx \rho(E)\,\Delta E$$

$$E = \frac{\hbar^2\pi^2}{2mL^2}\left(n_x^2 + n_{y_1}^2 + n_{z_1}^2 + n_{x_2}^2 + n_{y_2}^2 + n_{z_k}^2 + \dots\right) \qquad N(n)$$

$$n^2 = n_{x_1}^2 + n_{y_1}^2 + n_{z_1}^2 + n_{x_2}^2 + \dots)$$

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