From last time: Quantization of Resonator E+M field 24 Lecture

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$$\nabla \epsilon_0 \vec{E}(\vec{r},t) = 0$$

particle classical | Quantum 1st quantization | 2nd Quantization

pion nuclear physics

$$\frac{d}{dt}x(t) = \frac{p(t)}{m} \qquad \hat{H} = \frac{\hat{p}^2}{2m} + V \Rightarrow \hat{H} = \int \hat{\Psi}^+ \left(\frac{\hat{p}^2}{2m} + V\right)\hat{\Psi}\, d\vec{r}$$

$$\frac{d}{dt}p(t) = -\frac{dV}{dt}$$

$$\vec{\nabla} \times \vec{H} = \frac{\partial}{\partial t}\epsilon_0 \vec{E}$$

$$i\hbar\frac{\partial}{\partial t}\epsilon_0\vec{E} = i\hbar\,\nabla\times\vec{H} = -\hat{\vec{p}}\times\vec{H} \qquad i\hbar\frac{\partial}{\partial t}\begin{bmatrix}\epsilon_0\vec{E}\\ \mu_0\vec{H}\end{bmatrix} = \begin{bmatrix}0 & -\vec{p}\times\\ \vec{p}\times & 0\end{bmatrix}\begin{bmatrix}\vec{E}\\ \vec{H}\end{bmatrix}$$

$$i\hbar\frac{\partial}{\partial t}\mu_0\vec{H} = \hat{\vec{p}}\times\vec{E}$$

1st quantization not widely used but sometimes can give dramatically ~~dif~~ simplified.

$$\hat{\vec{p}} = \hat{p}_x\hat{i}_x + \hat{p}_y\hat{i}_y + \hat{p}_z i_z$$

Second Quantization

$$\hat{p} = -i\hbar\frac{\partial}{\partial x}$$

$$\frac{d}{dt}x(t) = \frac{p(t)}{m} \qquad \frac{d}{dt}e(t) = \frac{k}{\epsilon_0}h(t)$$

$$\hat{h} = -i\hbar A\frac{\partial}{\partial e}$$

$$\frac{d}{dt}p(t) = -m\omega_0^2 x(t) \qquad \frac{d}{dt}h(t) = \frac{-k}{\mu_0}e(t)$$

$$E = \frac{p^2(t)}{2m} + \frac{1}{2}m\omega_0^2 x^2 \qquad W = \frac{1}{2}\mu_0 L^3 h^2(t) + \frac{1}{2}\epsilon_0 L^3 e^2(t)$$

$$\hookrightarrow \hat{E}\psi(e,t) = \frac{1}{2}\mu_0 L^3 \hat{h}^2\psi(e,t) + \frac{1}{2}\epsilon_0 L^2 e^2\psi(e,t)$$

$$i\hbar\frac{\partial}{\partial t}\psi(e,t) = \qquad + \frac{1}{2}\epsilon_0 L^3 e^2\psi(e,t)$$

$$E\psi(e) = -\frac{1}{2}\mu_0 L^3 \hbar^2 A^2 \frac{d^2}{de^2}\psi(e) + \frac{1}{2}\varepsilon_0 L^3 e^2 \psi(e)$$

$$A = \frac{\omega_0 c}{L^3}$$

$$\hat{H} = -\frac{1}{2}\mu_0 L^3 \frac{\hbar^2 \omega_0^2 c^2}{L^6}\frac{d^2}{de^2} + \frac{1}{2}\epsilon_0 L^3 e^2 \longrightarrow -\frac{(\hbar\omega_0)^2}{2\epsilon_0 L^3}\frac{d^2}{de^2} + \frac{1}{2}\epsilon_0 L^3 e^2$$

$$i\hbar\frac{\partial}{\partial t}\psi(e,t) = -\frac{(\hbar\omega)^2}{2\epsilon_0 L^3}\frac{\partial^2}{\partial e^2}\psi(e,t) + \frac{1}{2}\epsilon_0 L^3 e^2\psi(e,t)$$

$e \rightarrow$ ~~energy of electric field~~. only includes time-dynamics of amplitude

$$\langle \hat{H} \rangle = \frac{1}{2}\mu_0 L^3 \langle \hat{h}^2 \rangle + \frac{1}{2}\epsilon_0 L^3 \langle e^2 \rangle$$

$$\phi_n(e) = \left[\frac{\epsilon_0 L^3}{\pi\hbar\omega_0}\right]^{1/4}\frac{1}{\sqrt{2^n n!}}\exp\left(\frac{-\frac{1}{2}\epsilon_0 L^3 e^2}{\hbar\omega_0}\right) H_n\left(\sqrt{\frac{\epsilon_0 L^3}{\hbar\omega}}\, e\right)$$

$$E_n = \hbar\omega_0\left(n + \frac{1}{2}\right)$$

$$\hat{e} = \sqrt{\frac{\hbar\omega_0}{2\epsilon_0 L^3}}\left(\hat{a} + \hat{a}^\dagger\right) \qquad \hat{h} = \sqrt{\frac{\hbar\omega_0}{2\mu_0 L^3}}\frac{(\hat{a} - \hat{a}^\dagger)}{i}$$

$$\hat{H} = \int \frac{1}{2}\mu_0 \hat{\vec{H}}(\vec{r}) + \hat{\vec{H}}(\vec{r}) + \frac{1}{2}\varepsilon_0 \hat{\vec{E}}(\vec{r}) \cdot \hat{\vec{E}}(\vec{r})\, d^3\vec{r}$$

$$= \frac{1}{2}\mu_0 L^3 \hat{h}^2 + \frac{1}{2}\epsilon_0 L^3 \hat{e}^2$$

$$= \hbar\omega_0\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right)$$

Multimoded Problem

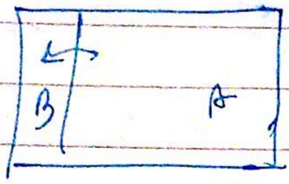
Classical $\vec{E}(\vec{r},t) = \sum_j e_j(t)\vec{u}_j(\vec{r}) \Rightarrow \frac{d}{dt}e_j(t) = \frac{k_j}{\epsilon_0}h_j(t)$

$$\vec{H}(\vec{r},t) = \sum_j h_j(t)\,\vec{V}_j(\vec{u}) \qquad \frac{d}{dt}h_j(t) = \frac{-k_j}{\mu_0}e_j(t)$$

$$\hat{H} = \sum_j \frac{1}{2}\mu_0 L^3 \hat{h}_j^2 + \frac{1}{2}\epsilon_0 L^3 e_j^2 \qquad \sum_j \frac{-(\hbar\omega_j)^2}{2\epsilon_0 L}\frac{d^2}{de_j^2} + \frac{1}{2}e_0 L^3 e_j^2$$

$E\Psi = \hat{H}\Psi \quad \Psi(e_1, e_2, \ldots) = \phi_n(e_1)\phi_{n_2}(e_2)$

$$E = \hbar\omega_1\left(n_1 + \frac{1}{2}\right) + \hbar\omega_2\left(n_2 + \frac{1}{2}\right) + \ldots$$



L

Also energy
dtn

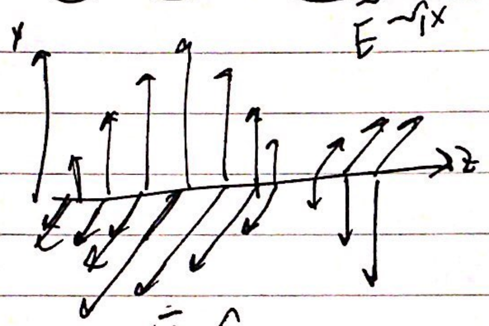
Energy density

Casimere effect

B < A < L

☆ metals w/ small separations → casimere forces... clamp metals together
(happens in)

Free-space version



$$\vec{E}(r,t) = e(t)\vec{u}(\vec{r})$$

$$\vec{H}(\vec{r},t) = h(t)\vec{v}(\vec{r})$$

$\Rightarrow \nabla\cdot\vec{u}=0$ $\nabla\times\vec{v}=ik\vec{u}$ $\frac{d}{dt}e(t) = \frac{-ik}{\epsilon_0}h(t)$

$\nabla\cdot\vec{v}=0$ $\nabla\times\vec{u}=ik\vec{v}$ $\frac{d}{dt}h(t) = \frac{-ik}{\mu_0}e(t)$

$e(t) \sim e^{\pm i\omega_0 t}$

$\omega_0 = ck$

$$\vec{E}(\vec{r},t) = \hat{i}_x e_0 e^{ikz} e^{-i\omega_0 t} + \text{Complex conjugate}$$

$$\vec{H}(\vec{r},t) = \hat{i}_y h_0 e^{ikz} e^{-i\omega_0 t} + \dots \quad \dots$$

$$W = \int \frac{1}{2}\epsilon_0 \vec{E}(r,t)\vec{E}(r,t) + \frac{1}{2}\mu_0 \vec{H}(\vec{r},t)\vec{H}(\vec{r},t)\, d^3r$$

$$E(\vec{r},t) = \hat{i}_x \sqrt{\frac{2\hbar\omega_0}{\epsilon_0 L^3}} \cos(kz-\omega_0 t) \quad \omega_0 = ck$$

$$H(\vec{r},t) = \hat{i}_y \sqrt{\frac{2\hbar\omega_0}{\mu_0 L^3}} \cos(kz-\omega_0 t)$$

1st Quantization

Work of Maxwell eqns
Clasp WEO soln's

$$\int \frac{1}{2}\epsilon_0\left(\vec{E}^{+}\cdot\vec{E}+\vec{E}\cdot\vec{E}^{+}\right)+\frac{1}{2}\mu_0\left(\vec{H}^{+}\cdot\vec{H}+\vec{H}\cdot\vec{H}^{+}\right)d_r^3=\hbar\omega_0$$

$$\vec{E}(\vec{r},t)\Big|_{1st\ quant}=\hat{i}_x\sqrt{\frac{\hbar\omega_0}{2\epsilon_0 L^3}}\,e^{ikz}e^{-i\omega_0 t}$$

$$H(\vec{r},t)\Big|_{1st\ quant}=\hat{i}_y\sqrt{\frac{\hbar\omega_0}{2\mu_0 L^3}}\,e^{ikz}e^{-i\omega_0 t}$$