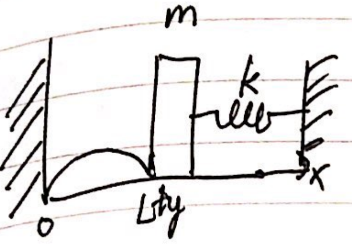


$$F = \frac{\Delta p}{\Delta t} = \frac{2\hbar k}{\Delta t} = \frac{\hbar^2 \pi^2 n^2}{mL^3}$$

$$\frac{\hbar^2 \pi^2 n^2}{mL^3} = k\Delta y_n$$

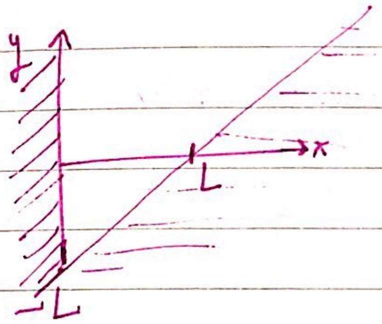
$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \quad + \frac{\hat{p}_y^2}{2M} + \frac{1}{2}ky^2$$

0 y+L

$$= \frac{\hat{p}_x^2}{2m} + \frac{\hat{p}_y^2}{2M} + V(x,y)$$

$$\omega_0^2 = \frac{k}{M}$$

$$\frac{1}{2}ky^2 = \frac{1}{2}M\omega_0^2 y^2$$



$$\chi(y) \sim e^{-\frac{M\omega_0 y^2}{2\hbar}}$$

The Adiabatic Approximation

$$\Psi(x,y) = \psi(y)\phi(x;y)$$

$$= \sum_n \sum_m c_{nm} \psi_n(y) \phi_m(x,y)$$

$$E_x(y)\phi(x,y) = \left(\frac{\hat{p}^2}{2m} + \quad \right)\phi(x,y)$$

0 L+y

$$\phi_n(x,y) = \sqrt{\frac{2}{L+y}} \sin\left(\frac{n\pi x}{L+n}\right) \qquad E_{x,n}(y) = \frac{\hbar^2 \pi^2}{2m(L+y)^2} n^2$$

$$E\psi(x,y) = \left(\frac{\hat{p}_x}{2m} + \quad + \frac{\hat{p}_y^2}{2M} + \frac{1}{2}M\omega_0^2 y^2\right)\psi(x,y)$$

0 L+y

$$E\psi(y)\phi(x,y) \approx \left(\frac{\hat{p}_x}{2m} + \quad + \frac{\hat{p}_y^2}{2M} + \frac{1}{2}M\omega_0^2 y^2\right]\psi(y)\phi(x,y)$$

0 L+y

$$\approx \left(E_x(y) + \frac{\hat{p}_y^2}{2M} + \frac{1}{2}M\omega_0^2 y^2\right]\psi(y)\,\phi(x,y))$$

$$\int_0^{L+y} \phi^*(x,y)\left|\left(E\,\psi(y)\,\phi(x,y) \approx \left(E_x(y) + \frac{\hat{p}_y^2}{2m} + \frac{1}{2}M\omega_0^2 y^2\right)\phi(x,y)\,\psi(y)\right)\right| dx$$

$$E\psi(y) \approx \left(\left\langle \phi(x,y)\right| \frac{-\hbar^2}{2m}\frac{\partial^2}{\partial y^2}\left|\phi(x,y)\right\rangle_x + E_x(y) + \frac{1}{2}M\omega_0^2 y^2\right)\psi(y)$$

$$E\psi(y) \approx \left(\frac{-\hbar^2}{2m}\frac{d^2}{dy^2} + \underbrace{E_x(y)}_{\frac{\hbar^2\pi^2 n^2}{2m(L+y)^2}} + \frac{1}{2}M\omega_0^2 y^2\right)\psi(y)$$

$$\frac{\hbar^2\pi^2}{2m(L+y)^2}n^2 \approx \frac{\hbar^2\pi^2}{2mL^2}n^2 - \frac{2\hbar^2\pi^2}{2mL^3}y + \dots$$

⇒ forced harmonic oscillator

$$y_0 = \frac{F_0}{M\omega_0^2}$$

$$E\psi(y) = \left(\frac{\hat{p}_y^2}{2M} + \frac{1}{2}M\omega_0^2 y^2 + \frac{\hbar^2\pi^2}{2mL^2}n^2 - \underbrace{\frac{\hbar^2\pi^2}{mL^3}y}_{-F_0 y}\right)\psi(y)$$

$$E = \hbar\omega_0\left(n+\frac{1}{2}\right) + \frac{\hbar^2\pi^2}{2mL^2}n_x^2 - \frac{F_0^2}{m\omega_0^2}$$

Quantization of Maxwell's Equations

Resonator		Free Space
classical	→	classical
first quantization	→	1st Quant
Second quantization	→	2nd Quant

Maxwell's eqns

$$\nabla\cdot\varepsilon_0\vec{E}(\vec{r},t) = 0$$

$$\nabla\cdot\mu_0 H(\vec{r},t) = 0$$

$$\nabla\times\vec{E}(\vec{r},t) = -\frac{\partial}{\partial t}\mu_0\vec{H}(\vec{r},t)$$

$$\nabla\times\vec{H}(\vec{r},t) = \frac{\partial}{\partial t}\epsilon_0\vec{E}(\vec{r},t)$$

Separate $\vec{r}, t$

$$\vec{E}(r,t) = e(t)\,\vec{u}(\vec{r})$$

$$\vec{H}(\vec{r},t) = h(t)\,\vec{v}(\vec{r})$$

$$\nabla\times\vec{u}(\vec{r}) = k\vec{v}(\vec{r})$$

$$\nabla\times\vec{v}(\vec{r}) = k\vec{u}(\vec{r})$$

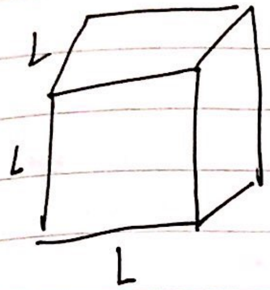
$$\nabla\cdot\vec{u}(\vec{r}) = 0$$

$$\nabla\cdot\vec{u}(\vec{r}) = 0$$

$$ke(t) = -\frac{\partial}{\partial t}\mu_0 h(t)$$

$$kh(t) = \frac{\partial}{\partial t}\epsilon_0 e(t)$$

transfer magnetic

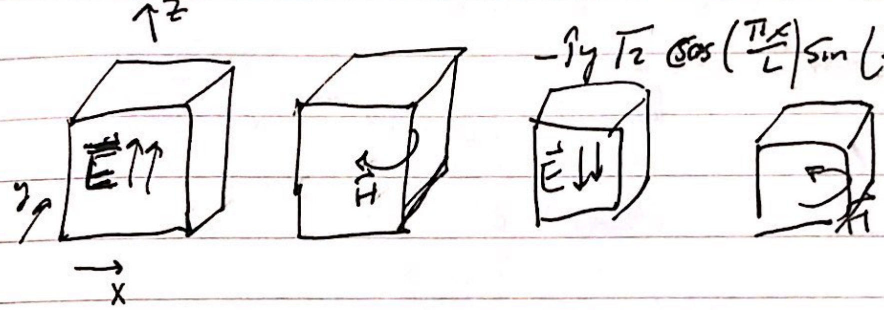


Lowest TM sol'n

$$\vec{u}(x,y,z) = \hat{1}_z \sqrt{2} \sin\left(\frac{\pi x}{L}\right) \sin\left(\frac{\pi y}{L}\right)$$

$$\vec{v}(x,y,z) = \hat{1}_x \sqrt{2} \sin\left(\frac{\pi x}{L}\right) \cos\left(\frac{\pi y}{L}\right)$$

$$-\hat{1}_y \sqrt{2} \cos\left(\frac{\pi x}{L}\right) \sin\left(\frac{\pi y}{L}\right)$$



$$\frac{d}{dt} e(t) = \frac{k}{\varepsilon_0} h(t)$$

$$\frac{d}{dt} h(t) = \frac{-k}{\mu_0} e(t)$$

$$\frac{d^2}{dt^2} e(t) = \frac{k}{\varepsilon_0} \frac{dh}{dt} = \frac{-k^2}{\varepsilon_0 \mu_0} e(t)$$

$$e(t) = \cos(\omega_0 t) \qquad \omega_0^2 = \frac{k^2}{\varepsilon_0 \mu_0} = c^2 k^2 \qquad c^2 = \frac{1}{\varepsilon_0 \mu_0}$$

Resonator energies

$$W = \int \frac{1}{2} \varepsilon_0 \vec{E}(\vec{r},t) \vec{E}(\vec{r},t) + \frac{1}{2} \mu_0 \vec{H}(\vec{r},t) \cdot \vec{H}(\vec{r},t) \, d^3\vec{r}$$

$$\vec{E}(\vec{r},t) = A \cos(\omega_0 t) \vec{u}(\vec{r}) \qquad W = \hbar\omega_0 = \int \frac{1}{2} \epsilon_0 A^2 \cos^2(\omega_0 t) \vec{u}(\vec{r}) \vec{u}(\vec{r})$$

$$\vec{H}(\vec{r},t) = B \sin(\omega_0 t) \vec{v}(\vec{r}) \qquad + \frac{1}{2} \mu_0 B^2 \sin^2(\omega_0 t) \vec{v}(\vec{r}) \vec{v}(\vec{r}) \, d^3\vec{r}$$

$$\int \vec{u}(\vec{r}) \cdot \vec{u}(\vec{r}) \, d^3\vec{r} = L^3$$

$$W = \frac{1}{2} \varepsilon_0 L^3 A^2 \cos^2(\omega_0 t) + \frac{1}{2} \mu_0 L^3 B^2 \sin^2(\omega_0 t) = \hbar\omega_0$$

$$\int \vec{v}(\vec{r}) \cdot \vec{v}(\vec{r}) \, d^3\vec{r} = L^3$$

$$= \frac{1}{2} \epsilon_0 L^3 A^2 = \hbar\omega_0 \Rightarrow A = \sqrt{\frac{2\hbar\omega_0}{\epsilon_0 L^3}} \quad B = -\sqrt{\frac{2\hbar\omega_0}{\mu_0 L^3}}$$

1-photon, classical solution

$$\vec{E}(\vec{r},t) = \sqrt{\frac{2\hbar\omega_0}{\epsilon_0 L^3}} \vec{u}(\vec{r}) \cos(\omega_0 t)$$

$$\vec{H}(\vec{r},t) = -\sqrt{\frac{2\hbar\omega_0}{\mu_0 L^3}} \vec{v}(\vec{r}) \sin(\omega_0 t)$$

Quantum Theory in 1st Quantization

Use maxwell's equations for schrödinger equation

$$\begin{cases} \nabla \cdot \varepsilon_0 \vec{E} = 0 \\ \nabla \cdot \mu_0 \vec{H} = 0 \\ \nabla \times \vec{E} = -\frac{\partial}{\partial t}\mu_0 \vec{H} \\ \nabla \times \vec{H} = \frac{\partial}{\partial t}\varepsilon_0 \vec{E} \end{cases}$$

$\Rightarrow$ use solutions but dump pieces with $\omega < 0$

Can't be a theory for photon in free space

$\longrightarrow$ you get negative frequencies (haven't found neg. energy photons yet)

$$\hat{\vec{E}}(r,t)\Big|_{1st\ quant.} = \sqrt{\frac{\hbar\omega_0}{2\varepsilon_0 L^3}}\ \vec{u}(\vec{r})\, e^{-i\omega_0 t}$$

$$\cos(\omega_0 t) = \frac{e^{i\omega_0 t} + e^{-i\omega_0 t}}{2}$$

$$\sin(\omega_0 t) = \frac{e^{i\omega_0 t} - e^{-i\omega_0 t}}{2i}$$

$$\vec{H}(\vec{r},t)\Big|_{1st\ quantization} = -i\sqrt{\frac{\hbar\omega_0}{2\mu_0 L}}\ \vec{V}(\vec{r})\, e^{-i\omega_0 t}$$

Normalization

$$\int \frac{1}{2}\varepsilon_0\left(\vec{E}^*(\vec{r},t)\vec{E}(\vec{r},t) + \vec{E}(\vec{r},t)\cdot\vec{E}^*(\vec{r},t)\right) + \frac{1}{2}\mu_0\left(\vec{H}^*(\vec{r},t)\vec{H}(\vec{r},t) + \vec{H}(\vec{r},t)\vec{H}^*(\vec{r},t)\right) d^3\vec{r} \Bigg|_{1st\ Quantization}$$

$$= \hbar\omega_0$$

2nd Quantization

$$\frac{d}{dt}e(t) = \frac{k}{\varepsilon}h(t) \qquad \frac{d}{dt}x(t) = \frac{p(t)}{m}$$

$$\frac{d}{dt}h(t) = \frac{-k}{\mu_0}e(t) \qquad \frac{d}{dt}p(t) = -m\omega_0^2 x(t)$$

$$E = \frac{1}{2}\varepsilon_0 L^3 e^2(t) + \frac{1}{2}\mu_0 L^3 h^2(t) \qquad E = \frac{p^2(t)}{2m} + \frac{1}{2}m\omega_0^2 x^2(t)$$