Lecture 22 10/28/19

From last time

$$\hat{H} = \left[\frac{\hat{p}^2}{2m} + V(x)\right] + \left[\frac{1}{2}L\hat{i}^2 + \frac{1}{2}Cv^2\right] + \frac{e}{d}vx \qquad V_0 = \frac{e}{d}\sqrt{\frac{\hbar}{2}\frac{\omega_0}{C}}\langle\phi_1|x|\phi_2\rangle$$

$$\frac{\hat{p}}{2m} + V(x) \rightarrow \hat{H}_{2\text{-level}}$$

$$\hat{H} = |\phi_1\rangle H_{11}\langle\phi_1| + |\phi_2\rangle H_{22}\langle\phi_2| + \left[\frac{1}{2}L\hat{i}^2 + \frac{1}{2}Cv^2\right] + \frac{e}{d}v\langle\phi_1|x|\phi_2\rangle \; (|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|)$$

$$\hat{H} = |\phi_1\rangle H_{11}\langle\phi_1| + |\phi_2\rangle H_{22}\langle\phi_2| + \hbar\omega_0\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) + V_0(\hat{a} + \hat{a}^\dagger)\,(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|)$$

$$\hat{H} = \hat{H}_A + \hat{H}_B + \hat{H}_{int}$$

looking ahead

→ what do you do when you have multiple degrees of freedom

→ quantization of an electromagnetic field

→ golden rule → continuum of states (density of states & statistical mechanics

→ hydrogen atom.

Problems in multiple dimensions

$$\hat{H} = \hat{H}_x(x) + \hat{H}_y(y)$$

$$E\Psi(x,y) = (\hat{H}_x(x) + \hat{H}_y(y))\Psi(x,y)$$

$$\Psi(x,y) = X(x)\,Y(y)$$

$$\frac{E\,X(x)\,Y(y)}{X(x)Y(y)} = \frac{\hat{H}_x(x)X(x)Y(y)}{X(x)\,Y(y)} + \frac{\hat{H}_y(y)X(x)Y(y)}{X(x)\,Y(y)}$$

$$E = \underbrace{\frac{\hat{H}_x(x)X(x)}{X(x)}}_{E_x} + \underbrace{\frac{\hat{H}_y(y)Y(y)}{Y(y)}}_{E_y}$$

Exam: put square well + LC circuit etc together into problem.

$$E_x X(x) = \hat{H}_x X(x)$$

$$E_y Y(y) = \hat{H}_y Y(y)$$

$$E = E_x + E_y$$

Example: $\hat{H} = \frac{|\hat{p}|^2}{2m} = \frac{\hat{p}_x^2}{2m} + \frac{\hat{p}_y^2}{2m} + \frac{\hat{p}_z^2}{2m}$

$$E\Psi(x,y,z) = \frac{|\hat{p}|^2}{2m}\Psi(x,y,z)$$

$$E_x X(x) = \frac{\hat{p}_x^2}{2m}X(x) = \frac{-\hbar^2}{2m}\frac{d^2}{dx^2}X(x)$$

$$X(x) = e^{ik_x x} \qquad E_x = \frac{\hbar^2 k_x^2}{2m}$$

$$\Psi(x,y,z) = e^{ik_x x}e^{ik_y y}e^{ik_z z} = e^{i\vec{k}\cdot\vec{r}}$$

$$E = \frac{\hbar^2|\vec{k}|^2}{2m}$$

$$i\hbar\frac{\partial}{\partial t}\Psi(x,y,t) = (\hat{H}_x+\hat{H}_y)\Psi(x,y,t)$$

$$\Psi(x,y,t) = X(x,t)\,Y(y,t)$$

$$i\hbar\frac{\partial}{\partial t}XY = \quad i\hbar\left(\frac{\partial X}{\partial t}\right)Y + i\hbar\left(\frac{\partial Y}{\partial t}\right)X = \hat{H}_x X Y + \hat{H}_y X Y$$

$$\underbrace{\frac{(i\hbar\frac{\partial}{\partial t}-\hat{H}_x)X}{X}}_{=C(t)} + \underbrace{\frac{(i\hbar\frac{\partial}{\partial t}-\hat{H}_y)Y}{Y}}_{-C(t)} = 0$$

$$i\hbar\frac{\partial}{\partial t}X(x,t) = \hat{H}_x(x)X(x,t) + \cancel{C(t)X(x,t)}^{\,0} \qquad \text{can take } C(t)\to 0$$

$$i\hbar\frac{\partial}{\partial t}Y(x,t) = \hat{H}_y(y)Y(x,t) - \cancel{C(t)Y(x,t)}^{\,0}$$

$$\hat{H} = \frac{|p|^2}{2m} + \frac{1}{2}m\omega_0^2(x^2+y^2) \qquad i\hbar\frac{\partial}{\partial t}\Psi(x,y,t) = \left[\frac{|\hat{p}|^2}{2m} + \frac{1}{2}m\omega_0^2(x^2+y^2)\right]\Psi(x,y,t)$$

$$i\hbar\frac{\partial}{\partial t}X(x,t) = \left(\frac{\hat{p}_x^2}{2m} + \frac{1}{2}m\omega_0^2x^2\right)X(x,t)$$

$$\Psi(x,y,z,t) = \left(\frac{m\omega_0}{\pi\hbar}\right)^{1/4} e^{-i\theta_x(t)} e^{iP_x(t)(x-X(t))} e^{-\frac{m\omega_0}{2\hbar}(x-X(t))^2}$$

$$\times\left(\frac{m\omega_0}{\pi\hbar}\right)^{1/4} e^{-i\theta_y(t)} e^{iP_y(t)(y-Y(t))} e^{-\frac{m\omega_0}{2\hbar}(y-Y(t))^2}$$

$$\times\frac{1}{(\pi L^2)^{1/4}}\frac{e^{ik_0z}e^{-i\frac{\hbar k_0^2 t}{2m}}}{\left(1+\frac{i\hbar t}{mL^2}\right)^{1/2}} e^{-\frac{\left(x-\frac{\hbar k_0 t}{m}\right)^2}{2L^2\left(1+\frac{i\hbar t}{mL^2}\right)}}$$

$$\epsilon\Psi(x,y) = \left[-\frac{\partial^2}{\partial x^2} - \frac{\partial^2}{\partial y^2} + x^2|y|\right]\Psi(x,y)$$

$$\epsilon_t = \frac{\langle\Psi_t(x,y)|\hat{H}|\Psi_t(x,y)\rangle}{\langle\Psi_t(x,y)|\Psi_t(x,y)\rangle} \qquad \Psi_t(x,y) = \left[\frac{2\alpha}{\pi}\right]^{1/4}e^{-\alpha x^2}\left[\frac{2\beta}{\pi}\right]^{1/4}e^{-\beta y^2}$$

$$= \alpha+\beta+\left(\frac{2}{\pi}\right)^{1/4}\frac{1}{4\alpha\beta^{3/4}} \qquad \frac{\partial}{\partial\alpha}\epsilon_t = 0 \quad \frac{\partial}{\partial\beta}\epsilon_t = 0$$

$$E\psi(x,y) = \left[\frac{\hat{p}_x^2}{2m} + \underset{\text{Ugly}}{V(x)}\right] + \left[\frac{\hat{p}_y^2}{2m} + \frac{1}{2}m\omega_0^2y^2\right] + \frac{1}{2}Kxy$$

$$\left[\frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_0^2y^2\right]\phi_n(y) = \hbar\omega_0\left(n+\tfrac{1}{2}\right)\phi_n(y) \qquad \Psi(x,y) = \sum_n \psi_n(x)\phi_n(y)$$

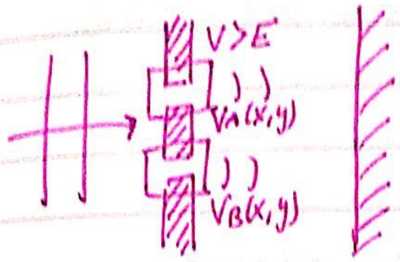
$$E\Psi(x,y) = (\hat{H}_x + \hat{H}_y + \hat{H}_{xy})\Psi(x,y)$$

$$\int_{-\infty}^{\infty}\phi_m(y)\left(E\sum_n \psi_n(x)\phi_n(y) = (\hat{H}_x + \hat{H}_y + \hat{H}_{xy})\sum_n \psi_n(x)\phi_n(y)\right)dy$$

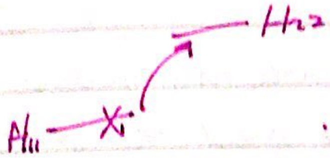
$$\langle\phi_n|\phi_m\rangle = \delta_{nm}$$

$$E\psi_n(x) = \hat{H}_x\psi_n(x) + \frac{1}{2}\hbar\omega_0\left(n+\frac{1}{2}\right)\psi_n(x) + \sum_n\langle\phi_m|\hat{H}_{xy}|\phi_n\rangle\psi_n(y)$$

multi-channel treatment/formalism.



$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{\hat{p}_y^2}{2m} + V(x,y) + \hat{H}^{(A)}_{2\text{-level}} + \hat{H}^{(B)}_{2\text{-level}}$$
$$+ V_A(x,y)\left(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|\right)_A$$
$$+ V_B(x,y)\left(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|\right)_B$$



this model is busted:
possibility of detecting in both A & B
→ need to fix the model.

$$\Phi_1 = \left(\frac{-}{*}\right)_A\left(\frac{-}{*}\right)_A$$
$$\Phi_2 = \left(\frac{*}{-}\right)_A\left(\frac{-}{*}\right)_B$$
$$\Phi_3 = \left(\frac{-}{*}\right)_A\left(\frac{*}{-}\right)$$

$$\hat{H} = \frac{\hat{p}_x^2}{2m} + \frac{\hat{p}_y^2}{2m} + V(x,y) + |\Phi_1\rangle(2H_{11})\langle\Phi_1| + |\Phi_2\rangle(H_{11}+H_{22})\langle\Phi_2| + |\Phi_3\rangle(H_{11}+H_{22})\langle\Phi_3|$$
$$+ V_A(x,y)\left(|\Phi_1\rangle\langle\Phi_2| + |\Phi_2\rangle\langle\Phi_1|\right)$$
$$+ V_B(x,y)\left(|\Phi_1\rangle\langle\Phi_3| + |\Phi_3\rangle\langle\Phi_2|\right)$$

2-level system

$$\Psi(x,y,\{\ \}) = \sum_{n=1}^{3}\Psi_n(x,y)|\Phi_n\rangle$$
$$= \psi(x,y)|\Phi_1\rangle + \Psi_2(x,y)|\Phi_2\rangle + \Psi_3(x,y)|\Phi_3\rangle$$

$$E\Psi_1(x,y) = \left[\frac{\hat{p}_x^2 + \hat{p}_y^2}{2m} + V(x,y)\right]\Psi_1(x,y) + V_A(x,y)\Psi_2(x,y) + V_B(x,y)\Psi_3(x,y)$$

$$E\Psi_2(x,y) = \left[\quad \dots \quad\right]\Psi_2(x,y) + V_A(x,y)\Psi_1(x,y)$$