Lecture 2nd

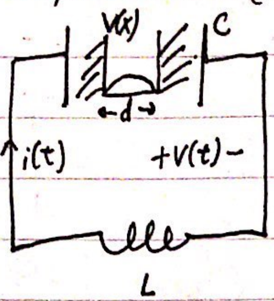
recap of stuff

$$\hat{H} = \hat{H}_{A} + \hat{H}_{B} + \hat{H}_{AB}$$

important

On final

From last time: Coupled Quantum Systems



$$\hat{H} = \underbrace{\frac{\hat{p}^2}{2m} + V(x)}_{\hat{H}_{particle}} + \underbrace{\frac{1}{2}L\hat{i}^2 + \frac{1}{2}CV^2}_{\hat{H}_{LC}} + \underbrace{\frac{e}{d}VX}_{H_{int}}$$

$$i\hbar\frac{\partial}{\partial t}\psi(x,v,t) = \hat{H}(x,v)\,\psi(x,v,t)$$

Capstone of 1st half of course

Ehrenfest theorem eq'ns

$$\frac{d}{dt}\langle x\rangle = \frac{\langle\hat{p}\rangle}{m}$$

$$\frac{d}{dt}\langle\hat{p}\rangle = -\left\langle\frac{dV}{dx}\right\rangle - \frac{e}{d}\langle v\rangle$$

Would also arise from

$$\hat{H}(t) = \frac{\hat{p}^2}{2m} + V(x) + \frac{e}{d}\langle v\rangle X$$

$$\frac{d}{dt}\langle v\rangle = \frac{1}{C}\langle\hat{i}\rangle$$

$$\frac{d}{dt}\langle\hat{i}\rangle = -\frac{1}{L}\langle v\rangle - \frac{1}{LC}\frac{e}{d}\langle x\rangle$$

Would also come from

$$\hat{H}(t) = \frac{1}{2}L\hat{i}^2 + \frac{1}{2}CV^2 + \frac{e}{d}\langle x\rangle V$$

Crude Derivations

independent particle approximation

$$i\hbar\frac{\partial}{\partial t}\Psi = (\hat{H}_A + \hat{H}_B + \hat{H}_{int})\Psi \qquad \Psi = \psi_A\psi_B \qquad i\hbar\frac{\partial}{\partial t}\psi_B = \hat{H}_B\psi_B$$

$$i\hbar\frac{\partial}{\partial t}\psi_A \approx \hat{H}_A\psi_A + \langle\psi_B|\hat{H}_{int}|\psi_B\rangle\psi_A$$

Change of basis?

Rotations (Pset)

one problem in pset does this

$$E\psi = \hat{H}\psi$$

$$\psi = e^{i\hat{S}}\phi$$

$$Ee^{i\hat{S}}\phi = \hat{H}e^{i\hat{S}}\phi$$

$$E\phi = \underbrace{e^{-i\hat{S}}\hat{H}e^{i\hat{S}}}_{\hat{H}'}\phi$$

$$\Longrightarrow \hat{H} = a\hat{\sigma}_z + b\hat{\sigma}_x$$

$$\hat{H} = \sqrt{a^2+b^2}\,\sigma_z$$

Dynamic case

$$i\hbar\frac{\partial}{\partial t}\psi = \hat{H}(t)\psi$$

$$\psi = e^{i\hat{S}(t)}\phi$$

$$i\hbar\frac{\partial}{\partial t}\psi = e^{i\hat{S}}\, i\hbar\frac{\partial\phi}{\partial t} - \hbar\frac{\partial\hat{S}}{\partial t}\phi$$

$$= \hat{H}e^{i\hat{S}}\phi$$

$$= i\hbar\frac{\partial}{\partial t}\phi = e^{-i\hat{S}}\hat{H}e^{i\hat{S}}\phi + \hbar\frac{\partial\hat{S}}{\partial t}\phi$$

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$$\hat{H}_{particle} = \frac{\hat{p}^2}{2m} + V(x) \longrightarrow |\phi_1\rangle H_{11} \langle\phi_1| + \cancel{|\phi_1\rangle H_{12}}\langle\phi_2|$$

$$+ |\phi_2\rangle \cancel{H_{21}} \langle\phi_1| + |\phi_2\rangle H_{22} \langle\phi_2|$$

E_1, E_2

$$\left[\frac{\hat{p}^2}{2m} + V(x)\right]\phi_j = E_j\phi_j \longrightarrow H_{kk} \Leftarrow E_j$$

$$H_{12} \langle\phi_1|\hat{H}_{part}|\phi_2\rangle$$

$$= \langle\phi_1|\frac{\hat{p}^2}{2m} + V(x)|\phi_2\rangle$$

$$= E_2\langle\phi_1|\phi_2\rangle = 0$$

$$\hat{H} \to |\phi_1\rangle H_{11}\langle\phi_1| + |\phi_2\rangle H_{12}\langle\phi_2|$$
$$+ \hbar\omega_0\left(\hat{a}^+\hat{a} + \tfrac{1}{2}\right)$$
$$+ \frac{e}{d}\langle\phi_1|x|\phi_2\rangle\left(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|\right)\sqrt{\frac{\hbar\omega_0}{2C}}\left(\hat{a}^+ + \hat{a}\right)$$

$$\hat{H} = |\phi_1\rangle H_{11}\langle\phi_1| + |\phi_2\rangle H_{22}\langle\phi_2| + \hbar\omega_0\left(\hat{a}^+\hat{a} + \tfrac{1}{2}\right)$$
$$+ V_0\left(\hat{a}^+ + \hat{a}\right)\left(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|\right)$$

Spin-boson model

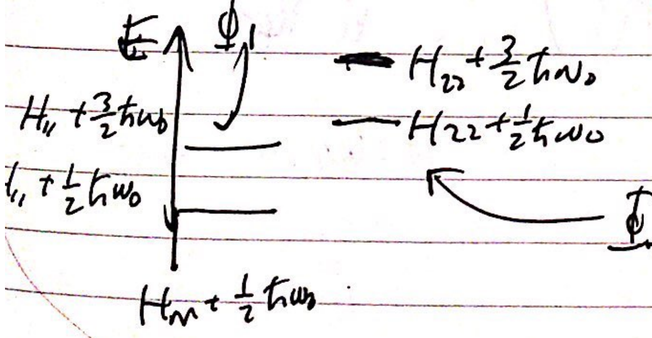
$$V_0 = \frac{e}{d}\langle\phi_1|x|\phi_2\rangle\sqrt{\frac{\hbar\omega_0}{2C}}$$

$$\hat{H}\sum_k C_k\left(C_1^{(k)}\phi_1 + C_2^{(k)}\phi_2\right)\left(\sum_n a_n^{(k)}|n\rangle\right)$$

Suppose $V_0 = 0$

$$E_{n,m} = \hbar\omega_0\left(n + \tfrac{1}{2}\right) \qquad \Phi_{nm} = |n\rangle|\phi_m\rangle$$

$$H_0\Phi_{nm} = \left(|\phi_1\rangle H_{12}\langle\phi| + |\phi_2\rangle H_{12}\langle\phi| + \hbar\omega_0\left(\hat{a}^+\hat{a} + \tfrac{1}{2}\right)\right)\Phi_{nm}$$
$$= E_{nm}\Phi_{nm}$$



Two level system model

$$\hat{H} = |\Phi_1\rangle\langle\Phi_1|\hat{H}|\Phi_1\rangle\langle\Phi_1| + |\Phi_1\rangle\langle\Phi_1|\hat{H}|\Phi_2\rangle\langle\Phi_2| + |\Phi_2\rangle\langle\Phi_2|\hat{H}|\Phi_1\rangle\langle\Phi_1|$$
$$+ |\Phi_2\rangle\langle\Phi_2|\hat{H}|\Phi_2\rangle\langle\Phi_2|$$

$$\langle\Phi_1|\hat{H}|\Phi_1\rangle = H_{11} + \tfrac{3}{2}\hbar\omega_0 \qquad \langle\Phi_1|\hat{H}|\Phi_2\rangle = ?$$

$$\langle\Phi_2|\hat{H}|\Phi_2\rangle = H_{22} + \tfrac{1}{2}\hbar\omega_0$$

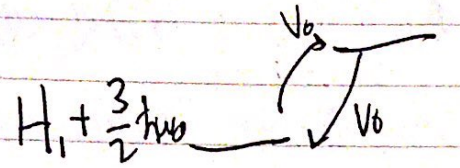
$$\langle\Phi_1|\hat{H}|\Phi_2\rangle = \langle n{=}0\rangle|\phi_1\rangle\Big| V_0\left(\hat{a}^+ + \hat{a}\right)\left(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|\right)|n=0\rangle\phi_2\rangle\rangle$$
$$= V_0\langle m=1|\hat{a}^+ + \hat{a}|n=0\rangle\left(\langle\phi_1|\left(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|\right)|\phi_2\rangle\right)$$

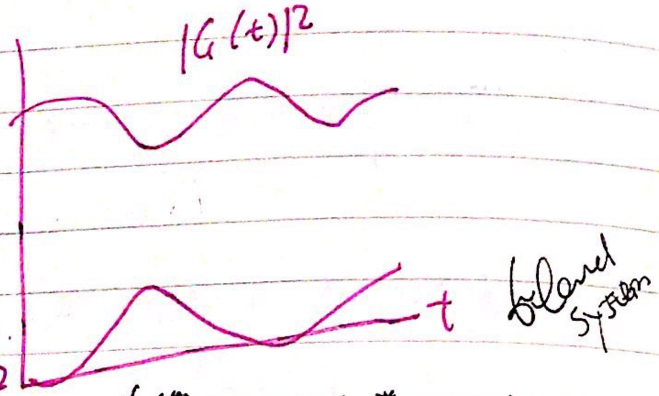
$V_0 =$

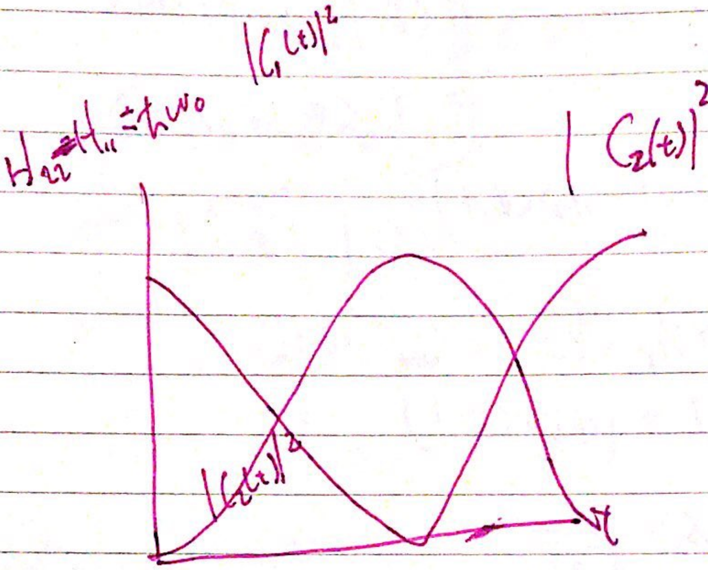
$$\langle n=1|\hat{a}^+|n=0\rangle = 1$$
$$+ \langle n=1|\hat{a}|n=0\rangle = 0$$

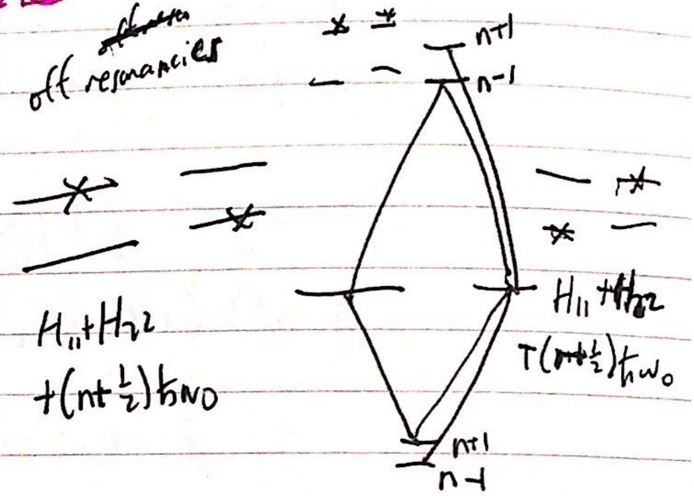
$$\langle\phi_1|\phi_1\rangle\langle\phi_2|\phi_2\rangle = 1$$

$\Psi = C_1(t)\Phi_1 + C_2(t)\Phi_2 \quad V_0 = \frac{e}{V}\langle H_1|x|H_2\rangle\sqrt{\frac{\hbar\omega_0}{2C}}$









$E\bar{C} = \bar{\bar{H}}\,\bar{C}$

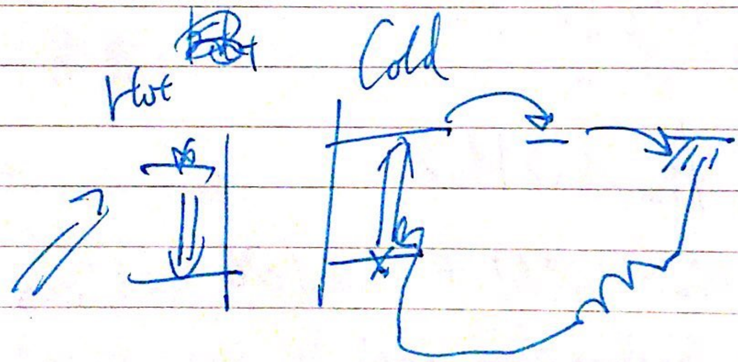
$EC_1 = (H_{11} + H_{22} + \frac{3}{2}\hbar\omega_0)C_1$

$+(\ )C_2$

$+(\ )C_3$

$+(\ )C_4$

$+(\ )C_5$



E linear $\neq C_2 - C_5$

$EC_1 + \Sigma_{11}(E)C_1 + H_{16}(E)C_6 + (H_{11} + H_{22} + \hbar\omega_0 \ldots(E))C_1$

$EC_6 = \Sigma_{66}(E)C_6 + H_{61}(E)C_1 + (\ )C_6$

No classical analog

$H_{16} = H_{16}(E)$

Research group does same thing w/ iron