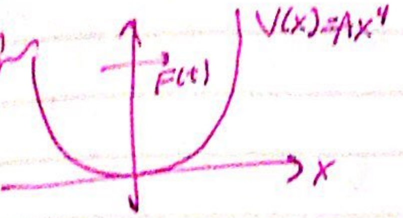
Lecture 20

Recall



$$\hat{H} = \underbrace{\frac{\hat{p}^2}{2m} + Ax^4}_{H_0} \underbrace{- F(t)x}_{V(t)}$$

$$\hat{H} = \sum_j \sum_k |\phi_j\rangle\langle\phi_j|\hat{H}|\phi_k\rangle\langle\phi_k|$$

$$\hat{H}_{2\text{-level}} = \sum_{j=1}^{2}\sum_{k=1}^{2} |\phi_j\rangle\langle\phi_j|\hat{H}|\phi_k\rangle\langle\phi_k|$$

$$= |\phi_1\rangle H_{11}\langle\phi_1| + |\phi_1\rangle H_{12}\langle\phi_2| + |\phi_2\rangle H_{21}\langle\phi_1| + |\phi_2\rangle H_{22}\langle\phi_2|$$

$$\hat{x}_{2\text{-level}} = \langle\phi_1|x|\phi_2\rangle\left(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|\right)$$

$$\frac{d}{dt}\langle\hat{x}_{2\text{-level}}\rangle = \frac{\langle\hat{p}_{2\text{-level}}\rangle}{m}$$

$$\hat{p} = m\omega_0\langle\phi_1|x|\phi_2\rangle\left(\frac{|\phi_1\rangle\langle\phi_2| - |\phi_2\rangle\langle\phi_1|}{i}\right)$$

$$= |\phi_1\rangle\langle\phi_1|\hat{p}|\phi_2\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_2|\hat{p}|\phi_1\rangle\langle\phi_1|$$

$$\frac{d}{dt}\langle\hat{p}_{2\text{-level}}\rangle = -m\omega_0^2\langle\hat{x}_{2\text{-level}}\rangle + \frac{2m\omega_0}{\hbar}\langle\phi_1|x|\phi_2\rangle^2 F(t)\underbrace{\langle|\phi_1\rangle\langle\phi_1| - |\phi_2\rangle\langle\phi_2|\rangle}_{(|c_1|^2 - |c_2|^2)}$$

$$\frac{2m\omega_0}{\hbar} = \frac{1}{\langle\phi_0|x|\phi_1\rangle^2_{SHO}} \qquad \frac{\langle\phi_0|x|\phi_1\rangle^2_{quartic}}{\langle\phi_0|x|\phi_1\rangle^2_{SHO}} = 0.99$$

matched $E_1 - E_0$

$$\langle X_{2\text{-level}}\rangle = \langle c_1\phi_1 + c_2\phi_2|\,\langle\phi_1|x|\phi_2\rangle\left(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1|\right)|c_1\phi_1 + c_2\phi_2\rangle$$

$$\Psi(x,t) = c_1(t)\phi_1(x) + c_2(t)\phi_2(x)$$

$$\rightarrow \langle\phi_1|x|\phi_2\rangle\langle c_1\phi_1 + c_2\phi_2|c_2\phi_1 + c_1\phi_2\rangle = \langle\phi_1|x|\phi_2\rangle\left(c_1^*c_2 + c_2^*c_1\right)$$

$$Q(t) = c_1^*(t)c_2(t) + c_2^*(t)c_1(t)$$

$$\langle\hat{P}_{2\text{-level}}\rangle = \langle c_1\phi_1 + c_2\phi_2|\,m\omega_0\langle\phi_1|x|\phi_2\rangle\left(\frac{|\phi_1\rangle\langle\phi_2| - |\phi_2\rangle\langle\phi_1|}{i}\right)|c_1\phi_1 + c_2\phi_2\rangle$$

$$= m\omega_0\langle\phi_1|x|\phi_2\rangle\left(\frac{c_1^*c_2 - c_2^*c_1}{i}\right)$$

$$P(t) = \frac{c_1^*(t)c_2(t) - c_2^*(t)c_1(t)}{i}$$

$$N(t) \leftarrow N^*(t) = |c_2(t)|^2 - |c_1(t)|^2$$

notation from chemistry meaning excited state

$$i\hbar\frac{d}{dt}\bar{c} + \frac{\bar{c}}{2T} = \bar{\bar{H}}\,\bar{c}$$

$$\frac{d}{dt}Q(t) = \omega_0 P(t) + \frac{2\,\mathrm{Im}\{H_{12}(t)\}}{\hbar}N(t)$$

$$\frac{d}{dt}P(t) = -\omega_0 Q(t) + \frac{2\,\mathrm{Re}\{H_{12}(t)\}}{\hbar}N(t)$$

$$\frac{d}{dt}N(t) = \frac{-2\,\mathrm{Re}\{H_{12}(t)\}}{\hbar}P(t) - \frac{2\,\mathrm{Im}\{H_{12}(t)\}}{\hbar}Q(t)$$

Bloch Equations

$$\leftrightarrow i\hbar\frac{d}{dt}\begin{bmatrix} c_1(t) \\ c_2(t)\end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22}\end{bmatrix}\begin{bmatrix} c_1(t) \\ c_2(t)\end{bmatrix}$$

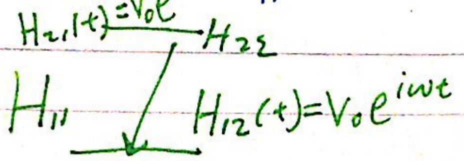
would give you a solution to this

$$\frac{d}{dt}Q(t) + \frac{Q(t)}{T_2} = -\omega_0 P(t) + \frac{2\,\mathrm{Im}\{H_{12}(t)\}}{\hbar} N(t)$$

$$\frac{d}{dt}P(t) + \frac{P(t)}{T_2} = -\omega_0 Q(t) + \frac{2\,\mathrm{Re}\{H_{12}(t)\}}{\hbar} N(t)$$

$$\frac{d}{dt}N(t) + \frac{N(t)-N_0}{T_1} = \frac{-2\,\mathrm{Re}\{H_{12}(t)\}}{\hbar} P(t)$$

⟸ optical bloch eq'ns (laser eq'ns)



$$\frac{d}{dt}Q + \frac{Q}{T_2} = \omega_0 P + \frac{2V_0 \sin(\omega t)}{\hbar} N$$

$$\frac{d}{dt}P + \frac{P}{T_2} = -\omega_0 Q + \frac{2V_0 \cos(\omega t)}{\hbar} N$$

Steady State

$$N \longrightarrow \frac{N_0}{1 + \frac{4V_0^2 T}{\hbar T_2}\frac{1}{(\omega-\omega_0)^2 + \frac{1}{T_2^2}}}$$

$$\Rightarrow N = \frac{N_0}{1 + \frac{I}{I_{sat}}}$$

$$\Downarrow\ Q = \frac{2V_0}{\hbar}\mathrm{Re}\left\{\frac{e^{i\omega t}}{\omega_0 - \omega + \frac{i}{T_2}}\right\} N$$

$$P = \frac{2V_0}{\hbar}\mathrm{Im}\left\{\frac{e^{i\omega t}}{\omega_0 - \omega + \frac{i}{T_2}}\right\} N$$

$$\frac{d}{dt}N + \frac{N}{T_1} = \frac{-4V_0^2}{\hbar^2 T_2}\frac{1}{(\omega-\omega_0)^2 + \frac{1}{T_2^2}} N$$

$$\langle \hat{x}_{2\text{-level}} \rangle = \langle \phi_1 | x | \phi_2 \rangle = (c_1^* c_2 + c_2^* c_1) = \langle \phi_1 | x | \phi_2 \rangle Q(t)$$

$$\hat{H}_{2\text{-level}} = |\phi_1\rangle H_{11} \langle \phi_2| + |\phi_1\rangle H_{12} \langle \phi_2|$$
$$+ |\phi_2\rangle H_{21} \langle \phi_1| + |\phi_2\rangle H_{22} \langle \phi_2|$$

$$\psi = c_1\phi_1 + c_2\phi_2$$

$$i\hbar\frac{d}{dt}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$ ⟵ $\hat{H}_{2\text{-level}}$

$$Q(t) = [c_1(t)\ c_2(t)]^* \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \langle \hat{\sigma}_x \rangle = c_1^* c_2 + c_2^* c_1$$

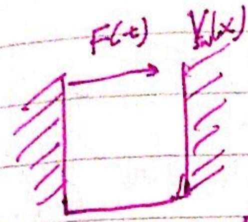
$$\hat{H} = -\frac{\Delta E}{2}\hat{\sigma}_z + V(t)\hat{\sigma}_x$$

$$P(t) = \frac{c_1^* c_2 - c_2^* c_1}{i} = [c_1\ c_2]^* \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \langle \hat{\sigma}_y \rangle$$

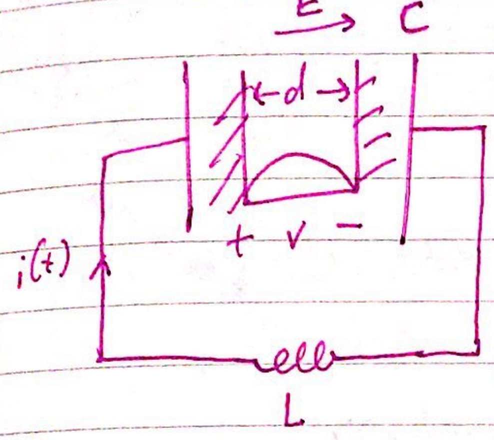
$$\hat{H} = \frac{\Delta E}{2}\hat{\sigma}_z + V(t)\hat{\sigma}_x$$

$$-N(t) = |c_1|^2 - |c_2|^2 = [c_1\ c_2]^* \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \langle \hat{\sigma}_z \rangle$$

↗ $\hat{\sigma}_z$



$$i\hbar\frac{\partial}{\partial t}\psi = \left[\frac{\hat{p}^2}{2m} + V_{sw}(x) - F(t)x\right]\psi(x,t)$$



$$\hat{H} = \left[\frac{\hat{p}^2}{2m} + V_{sw}(x)\right] + \left[\frac{1}{2}L\hat{i}^2 + \frac{1}{2}Cv^2\right] + \frac{evx}{d}$$

$$= \hat{H}_{particle} + \hat{H}_{LC} + \hat{H}_{int}$$ ↑ coupling term

charge $q = -|e| = -e$

$F = qE$

$= -eE$

$= -\frac{ev}{d}$

not sure what this is?

$$i\hbar\frac{\partial}{\partial t}\psi(x,v,t) = \left\{-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V_{sw}(x) - \frac{\hbar^2\omega_0^2}{2C}\frac{\partial^2}{\partial v^2} + \frac{1}{2}Cv^2 + \frac{evx}{d}\right\}\psi(x,v,t)$$

$P(x,v,t) = |\psi(x,v,t)^2|$

Is energy conserved?

$\frac{d}{dt}\langle\hat{H}\rangle = \frac{1}{i\hbar}\langle[\hat{H},\hat{H}]\rangle \quad \langle\frac{\partial}{\partial t}\hat{H}\rangle = 0$

$[\hat{H},\hat{H}] = 0$

$\frac{d}{dt}\langle x\rangle = \frac{1}{i\hbar}\langle[x,\hat{H}]\rangle = \frac{\langle\hat{p}\rangle}{m}$

$\frac{d}{dt}\langle v\rangle = \frac{1}{C}\langle\hat{i}\rangle$

$\frac{d}{dt}\langle\hat{p}\rangle = -\langle\frac{dV}{dx}\rangle + \langle F\rangle$

$\frac{d}{dt}\langle\hat{i}\rangle = -\frac{1}{L}\langle v\rangle - \frac{1}{LC}\frac{e}{d}\langle x\rangle$

$\langle F\rangle \leftarrow -\frac{e}{d}\langle v\rangle$

$\langle x\rangle = \langle\psi(x,v,t)|x|\psi(x,v,t)\rangle$

$= \int_{-\infty}^{\infty}dx\int_{-\infty}^{\infty}dv\,\{\psi^*(x,v,t)\,x\,\psi(x,v,t)\}$

$i\hbar\frac{\partial}{\partial t}\psi_{part}(x,t) = \left[\frac{\hat{p}^2}{2m} + V_{sw}(x)\right]\psi_{part}(x,t) + \frac{e}{d}\langle\psi_{LC}(v,t)|v|\psi_{LC}(v,t)\rangle\psi_{part}(x,t)$

$F(t) = -\frac{e}{d}\langle\psi_{LC}(v,t)|v|\psi_{LC}(v,t)\rangle$

$i\hbar\frac{\partial}{\partial t}\psi(x,v,t) = [\hat{H}_{particle} + \hat{H}_{LC} + \hat{H}_{int}(x,v)]\psi(x,v,t)$

$i\hbar\frac{\partial}{\partial t}\Psi = |\hat{H}_A + \hat{H}_B + \hat{H}_{int}|\Psi$

$\Psi \approx \psi_A\psi_B \qquad i\hbar\frac{\partial}{\partial t}\Psi = i\hbar\frac{\partial}{\partial t}\psi_A\psi_B = i\hbar\psi_B\frac{\partial}{\partial t}\psi_A + i\hbar\psi_A\frac{\partial}{\partial t}\psi_B$

If B is "Big" $\qquad = \hat{H}_A\psi_A\psi_B + \hat{H}_B\psi_A\psi_B + \hat{H}_{int}\psi_A\psi_B$

$i\hbar\frac{\partial}{\partial t}\psi_B \approx \hat{H}_B\psi_B \qquad \psi_A(i\hbar\frac{\partial}{\partial t}\psi_B - \hat{H}_B\psi_B) + i\hbar\psi_B\frac{\partial}{\partial t}\psi_A$

$\langle\psi_A|\psi_B\rangle = 1 \qquad \approx \psi_B\hat{H}_A\psi_A + \hat{H}_{int}\psi_A\psi_B \quad \rightarrow \quad \langle\psi_B|i\hbar\psi_B\frac{\partial}{\partial t}\psi_A \approx \psi_B\hat{H}_A\psi_A + \hat{H}_{int}\psi_A\psi_B)$

$$\boxed{i\hbar\frac{\partial}{\partial t}\psi_A \approx \hat{H}_A\psi_A + \langle\psi_B|\hat{H}_{int}|\psi_B\rangle\psi_A}$$