6.728 Lecture 19 10/21/19

$$\hat{H} = \underbrace{\frac{\hat{p}^2}{2m} + Ax^4}_{\hat{H}} \underbrace{- F(t)x}_{\hat{V}} \qquad \hat{H}_{2\text{-level}} = \left(|\phi_1\rangle H_{11} \langle\phi_1| + |\phi_1\rangle H_{12} \langle\phi_2| + |\phi_2\rangle H_{21} \langle\phi_1| + |\phi_2\rangle H_{22} \langle\phi_2| \right)$$

$$E_j \phi_j = \hat{H}_0 \phi_j$$

$$X_{2level} = \langle\phi_1|x|\phi_2\rangle \left(|\phi_1\rangle\langle\phi_2| + |\phi_2\rangle\langle\phi_1| \right)$$

$$\frac{d}{dt}\langle X_{2level}\rangle = \frac{\langle \hat{p}_{2level}\rangle}{m} \qquad \hat{P}_{2\text{-level}} = m\langle\phi_1|x|\phi_2\rangle \omega_0 \left(\frac{|\phi_1\rangle\langle\phi_2| - |\phi_1\rangle\langle\phi_2|}{i} \right)$$

$$\frac{d}{dt}\langle \hat{p}_{2\text{-level}}\rangle = -m\omega_0^2 \langle X_{2\text{-level}}\rangle + \frac{2m\omega_0}{\hbar}\langle\phi_1|x|\phi_2\rangle^2 F(t) \left(|c_1|^2 - |c_2|^2 \right)$$

Quartic Model

$$\frac{d}{dt}\langle\hat{x}\rangle = \frac{\langle\hat{p}\rangle}{m}$$

$$\frac{d}{dt}\langle\hat{p}\rangle = -\langle 4Ax^3\rangle + F(t)$$

2-level system:

$$\frac{d}{dt}\langle X_{2\text{-level}}\rangle = -\langle 4Ax^3\rangle + F(t)$$

$$\frac{d}{dt}\langle\hat{P}_{2\text{-level}}\rangle = -m\omega_0^2\langle X_{2level}\rangle + \frac{\langle\phi_1|x|\phi_2\rangle^2}{\langle\phi_1|x|\phi_2\rangle^2_{SHO}} F(t) \underbrace{\left(|c_1|^2 - |c_2|^2\right)^2}$$

if $|c_1|^2 \approx 1$

Forced SHO

$$\frac{d}{dt}\langle x\rangle = \frac{\langle\hat{p}\rangle}{m}$$

$$\frac{d}{dt}\langle\hat{p}\rangle = -m\omega_0^2\langle x\rangle + F(t)$$

Restoring force

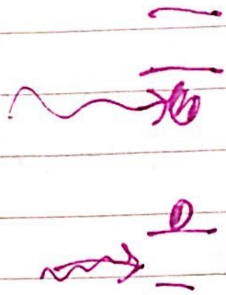
$-\langle 4Ax^3\rangle$ quartic well

$-m\omega_0^2\langle x\rangle$ 2-level system

$$\Psi(x,t) = c_1(t)\phi_1(x) + c_2(t)\phi_2(x) \Rightarrow \langle x\rangle = \langle\Psi|x|\Psi\rangle = \langle\phi_1|x|\phi_2\rangle(c_1^* c_2 + c_2^* c_1)$$

$$\approx 4A\langle\phi_1|x^3|\phi_2\rangle(c_1^* c_2 + c_2^* c_1)$$

$$-4A\langle\phi_1|x^3|\phi_2\rangle = -m\omega_0^2\langle\phi_1|x|\phi_2\rangle$$



goes to excited state

next atom pushes back so field is stronger

∴ Random laser stuff were diverse

$F(t)\left(|C_1|^2-|C_2|^2\right)$ push back on whoever's pushing it.

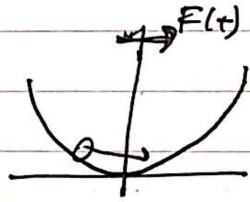
amplify vibrations

⇒ make lasers.

$|C_2|^2>|C_1|^2$

⇒ population inversion

Add loss, friction



$$\hat{H}=|\phi_1\rangle H_{11}\langle\phi_1|+|\phi_1\rangle H_{12}\langle\phi_2|+|\phi_2\rangle H_{21}\langle\phi_1|+|\phi_2\rangle H_{22}\langle\phi_2|$$

$$\frac{d}{dt}x(t)=\frac{p(t)}{m}$$

$$\frac{d}{dt}p(t)+\underbrace{\frac{p(t)}{\tau}}_{\text{decay for friction in classical model}}=-m\omega_0^2x(t)+F(t)$$

$$\frac{d}{dt}\langle X_{2\text{-level}}\rangle=\frac{\langle\hat{p}_{2\text{-level}}\rangle}{m}$$

$$\frac{d}{dt}\langle P_{2\text{-level}}\rangle+\frac{\langle\hat{p}_{2\text{-level}}\rangle}{\tau}=-m\omega_0^2\langle X_{2\text{-level}}\rangle+(\quad)F(t)$$

2 level model also used for spin systems