Lecture

10/18

From last time : Finite basis approx

$$\psi_t = \sum_i c_i \psi_i(x)$$

also works for ~~time~~ independent case $i\hbar \frac{d}{dt}\vec{c}(t) = \hat{H}\vec{c}$

$$E_t = \frac{\langle \psi_t | \hat{H} | \psi_t \rangle}{\langle \psi_t | \psi_t \rangle}$$

$$\frac{\partial E_t}{\partial c_1} = 0 \quad \frac{\partial E_t}{\partial c_2} = 0 \implies E_t \begin{bmatrix} c_1 \\ c_2 \\ \vdots \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} & \dots \\ & & \\ & & \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \end{bmatrix}$$

$$E_t \vec{c} = \hat{H} \cdot \vec{c}$$

2-level system : Approximate Hamiltonian

consider $\hat{H} = \hat{H}_0 + V(t) = \underbrace{\frac{\hat{p}^2}{2m} + Ax^4}_{H_0} \underbrace{- F(t)x}_{V(t)}$

in general $\hat{H} = \sum_i \sum_k |\phi_i\rangle\langle\phi_j|\hat{H}|\phi_k\rangle\langle\phi_k|$ (need ϕ_j to be complete)

$$\hat{H}_{2\text{-level}} = \sum_{j=1}^{2}\sum_{k=1}^{2} |\phi_j\rangle\langle\phi_j|\hat{H}|\phi_k\rangle\langle\phi_k| = \sum_{j=1}^{2}\sum_{k=1}^{2} |\phi_j\rangle H_{jk} \langle\phi_k|$$

Static Initial value problem

$t=0$ $\quad \psi(x, t=0) = \phi_i(x)$ ← without perturbation $V=0$

including V

eigenfunction expansion

$$\psi(x,t) = \sum_j a_j \phi_j e^{-\frac{iE_j t}{\hbar}} \text{ in general}$$

$$a_+ \phi_+(x) e^{\frac{-iE t}{\hbar}} \;\#\; a_- \phi_-(x) e^{\frac{-iE t}{\hbar}}$$

$$i\hbar \frac{d}{dt}\vec{c}(t) = \hat{H}(t)\vec{c}(t)$$

$$\vec{c}(t) = e^{\frac{\int \hat{H}(t)dt}{i\hbar}} \cdot \vec{c}(0)$$

More Statics prob

$$E\begin{bmatrix} C_1 \\ C_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}\begin{bmatrix} C_1 \\ C_2 \end{bmatrix}$$

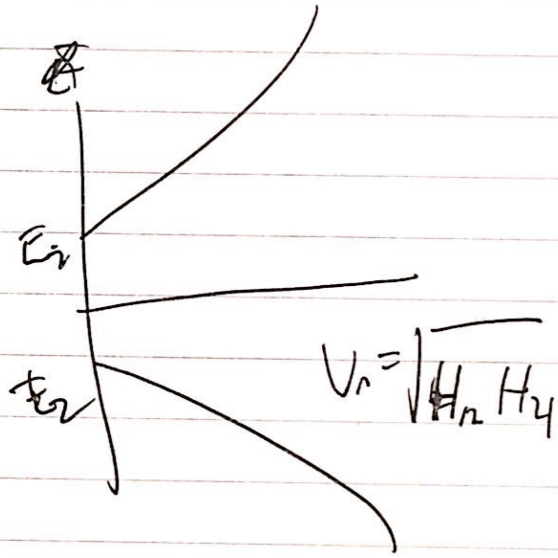
$$(H_{11}-E)(H_{22}-E) - H_{12}H_{21} = 0$$

$$E_\pm = \frac{H_{11}+H_{12}}{2} \pm \frac{1}{2}\sqrt{(H_{22}-H_{11})^2 + 4H_{12}H_{21}}$$

$$E\vec{c} = \vec{H}\cdot\vec{c}$$

$$(\vec{H}\cdot E\,\vec{\vec{I}})\cdot\vec{c} = 0$$

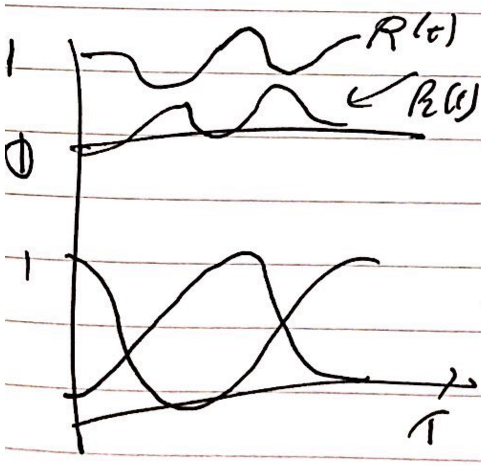
$$\det(\vec{H}\cdot E\,\vec{\vec{I}}) = 0$$

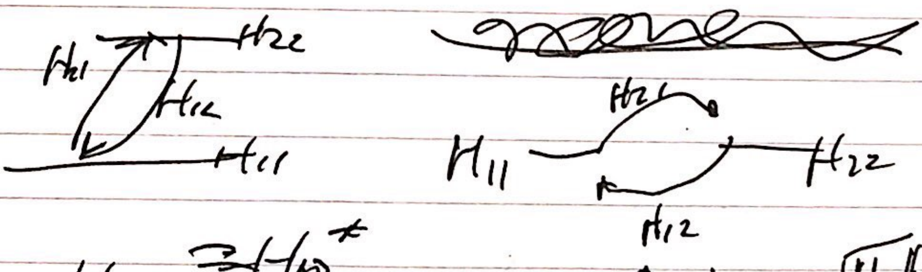


if $\sqrt{H_{12}H_{21}} \ll H_{22}-H_{11}$

$$E_+ \longrightarrow H_{22} + \frac{H_{21}H_{12}}{H_{22}-H_{11}} + $$

$$E_- \longrightarrow H_1 - \frac{H_{21}H_{12}}{H_{22}-H_1} + \cdots$$





$$H_{21} = H_{12}^*$$

$$P_2(t) \rightarrow \sin^2\left(\frac{\Omega t}{2}\right) = \sin^2\frac{\sqrt{H_{21}H_{12}}\,t}{\hbar}$$

$$= \sin^2(\Omega t)$$

Dynamic Problem

$$i\hbar\frac{\partial}{\partial t}\psi = \left[\frac{p^2}{2m} + Ax^4 - f(t)x\right]\psi$$

$$\hat{H}_{2\,level} = |\phi_1\rangle H_{11}\langle\phi_1| + |\phi_1\rangle H_{12}\langle\phi_2|$$

$$+ |\phi_2\rangle H_{21}\langle\phi_1| + |\phi_2\rangle H_{22}\langle\phi_2|$$

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