10/7 Squeezed States Lecture 14

$$i\hbar\frac{\partial}{\partial t}\psi(x,t)=\frac{-\hbar^2}{2m(t)}\frac{\partial^2}{\partial x^2}\psi(x,t)+\frac{1}{2}m(t)\omega_0^2x^2\psi(x,t)$$
$$-F(t)\,x\,\psi(x,t)+v(t)\,i\hbar\frac{\partial}{\partial x}\psi(x,t)$$

$\hookrightarrow -v(t)\hat{p}\psi$

$$\psi(x,t)=\left(\frac{2\alpha(t)}{\pi}\right)^{1/4}e^{-i\theta(t)}\,e^{\frac{ip(t)(x-X(t))}{\hbar}}\,e^{-\alpha(t)(x-X(t))^2}$$

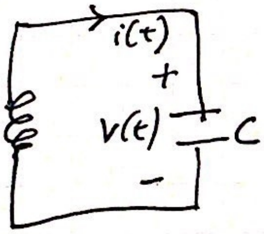
$$\frac{d}{dt}X(t)=\frac{p(t)}{m(t)}-v(t)$$

$$\frac{d}{dt}p(t)=-m(t)\omega_0^2(t)X(t)+F(t)$$

$$\hbar\frac{d}{dt}\theta(t)=\frac{\hbar^2\alpha(t)}{2m(t)}+\frac{m(t)\omega_0^2(t)}{8\alpha(t)}-\frac{p^2(t)}{2m(t)}+\frac{1}{2}m(t)\omega_0^2(t)X(t)-F(t)X(t)$$

$$i\hbar\frac{d}{dt}\alpha(t)=\frac{2\hbar^2\alpha^2(t)}{m(t)}-\frac{1}{2}m(t)\omega_0^2(t)$$

LC Circuit



$$\frac{d}{dt}v(t)=\frac{1}{C}i(t)$$

$$\frac{d}{dt}i(t)=-\frac{1}{L}v(t)$$

$$E=\frac{1}{2}Li^2(t)+\frac{1}{2}Cv^2(t)$$

SHO

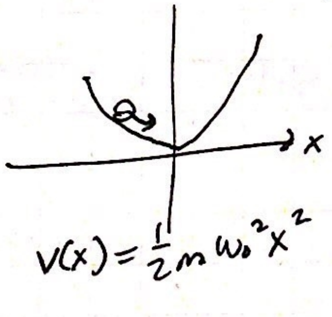
$$\frac{d}{dt}x(t)=\frac{p(t)}{m}$$

$$\frac{d}{dt}p(t)=-m\omega_0^2x(t)$$

$$E=\frac{p^2(t)}{2m}+\frac{1}{2}m\omega_0^2x^2(t)$$

$v(t)\leftrightarrow x(t)$

$i(t)\leftrightarrow p(t)$



$$V(x)=\frac{1}{2}m\omega_0^2x^2$$

$$E=\frac{p^2(t)}{2m}+\frac{1}{2}m\omega_0^2x(t)$$

$\hookrightarrow \hat{E}\psi(x,t)=\frac{\hat{p}^2}{2m}\psi(x,t)+\frac{1}{2}m\omega_0^2x^2\psi(x,t)$

$\hookrightarrow i\hbar\frac{\partial}{\partial t}\psi(x,t)=\frac{(-i\hbar\frac{\partial}{\partial x})^2}{2m}\psi(x,t)+\frac{1}{2}m\omega_0^2x^2\psi(x,t)$

$$E=\frac{1}{2}Li^2(t)+\frac{1}{2}Cv^2(t)$$

$\hookrightarrow \hat{E}\psi(v,t)=\frac{1}{2}L\hat{i}^2\psi(v,t)+\frac{1}{2}Cv^2\psi(v,t)$

$\hookrightarrow i\hbar\frac{\partial}{\partial t}\psi(v,t)=\frac{1}{2}L\left(-iA\frac{\partial}{\partial v}\right)^2\psi(v,t)+\frac{1}{2}Cv^2\psi(v,t)$

$$A=\omega_0^2=\frac{1}{LC}$$

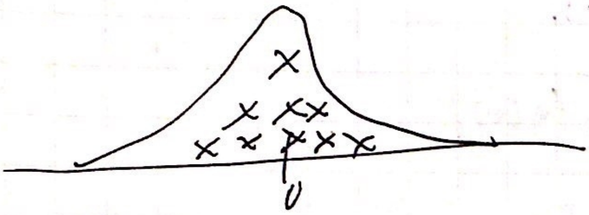
$$\boxed{\hat{H} = -\frac{\hbar^2\omega_0^2}{2C}\frac{\partial^2}{\partial v^2} + \frac{1}{2}Cv^2}$$

quantum mechanical soln for LC circuit.

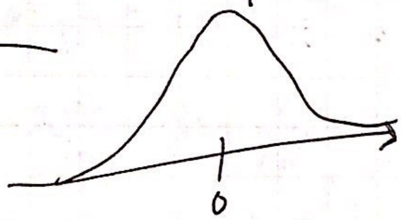
$$E\psi(v) = -\frac{\hbar^2\omega_0^2}{2C}\frac{d^2}{dv^2}\psi(v) + \frac{1}{2}Cv^2\psi(v)$$

$$\phi_n(v) = \left[\frac{C}{\pi\hbar\omega_0}\right]^{1/4}\frac{1}{\sqrt{2^n n!}}\, e^{-\frac{\frac{1}{2}Cv^2}{\hbar\omega_0}}\, H_n\left(\sqrt{\frac{C}{\hbar\omega_0}}\,v\right)$$

$$E_n = \hbar\omega_0\left(n+\tfrac{1}{2}\right)$$



$\rho(v) = |\phi_0(v)|^2$



Ground State

$$\int_{-\infty}^{\infty}\psi^*_{(v)}\left(E\psi(v) = \tfrac{1}{2}L\hat{i}^2\psi(v) + \tfrac{1}{2}Cv^2\psi(v)\right)dv$$

$$E = \frac{1}{2}L\langle\hat{i}^2\rangle + \frac{1}{2}C\langle v^2\rangle = \frac{1}{2}\hbar\omega_0$$

$$\langle v^2\rangle = \overbrace{\langle v\rangle^2}^{0} + (\Delta v)^2$$

$$\langle\hat{i}^2\rangle = \cancel{\langle\hat{i}\rangle^2}^{0} + (\Delta i)^2$$

$$= \frac{1}{2}L(\Delta i)^2 + \frac{1}{2}C(\Delta v)^2$$

$$\frac{1}{2}C(\Delta v)^2 = \frac{1}{4}\hbar\omega_0 \Rightarrow (\Delta v)^2 = \frac{1}{2}\frac{\hbar\omega_0}{C}$$

$$\frac{1}{2}L(\Delta i)^2 = \frac{1}{4}\hbar\omega_0 = (\Delta i)^2 = \frac{1}{2}\frac{\hbar\omega_0}{L}$$

$$\Delta v\Delta i = \sqrt{\frac{1}{2}\frac{\hbar\omega_0}{C}}\cdot\sqrt{\frac{1}{2}\frac{\hbar\omega_0}{L}} = \frac{1}{2}\frac{\hbar\omega_0}{\sqrt{LC}}$$

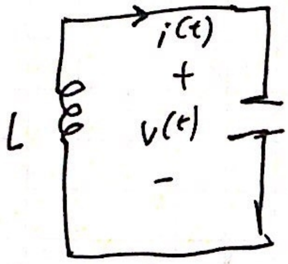
Gaussian ground state

$$\Delta v\Delta i \geq \frac{1}{2}\frac{\hbar\omega_0}{\sqrt{LC}}$$

$$\Rightarrow V = \sqrt{\frac{\hbar\omega_0}{2C}}\left(\hat{a} + \hat{a}^{+}\right)$$

$$\hat{i} = \sqrt{\frac{\hbar\omega_0}{2L}}\left(\frac{\hat{a} - \hat{a}^{+}}{i}\right)$$

$$\langle \phi_{n}(v) | v | \phi_n(v) \rangle = \sqrt{\frac{\hbar\omega_0}{2C}}\left(\sqrt{n}\,\delta_{n',n-1} + \sqrt{n+1}\,\delta_{n',n+1}\right)$$



$$V(t) = -L\frac{d}{dt}i(t)$$

$$i(t) = \frac{d}{dt}q(t)$$

$$V(t) = \frac{d}{dq}E_q(q(t))$$

From pset

* Don't know ground state energy

linear inductor, nonlinear capacitor $E = \frac{1}{2}Li^2(t) + E_q(q(t))$

$$\frac{d}{dt}q(t) = i(t) \qquad \frac{d}{dt}x(t) = \frac{p(t)}{m}$$

$$\frac{d}{dt}i(t) = -\frac{1}{L}\frac{d}{dq}E_q(q(t)) \qquad \frac{d}{dt}p(t) = -\frac{d}{dx}V(x(t))$$

$$E = \frac{1}{2}Li^2(t) + E_q(q(t)) \qquad E = \frac{p^2(t)}{2m} + V(x(t))$$

$q(t) \longleftrightarrow x(t)$

$i(t) \longleftrightarrow p(t)$

$$E = \frac{1}{2}Li^2(t) + E_q(q(t))$$

$$\hat{E}\psi(q,t) = \frac{1}{2}L\hat{i}^2\psi(q,t) + E_q(q)\psi(q,t)$$

dunno what to choose for $\hat{i}$

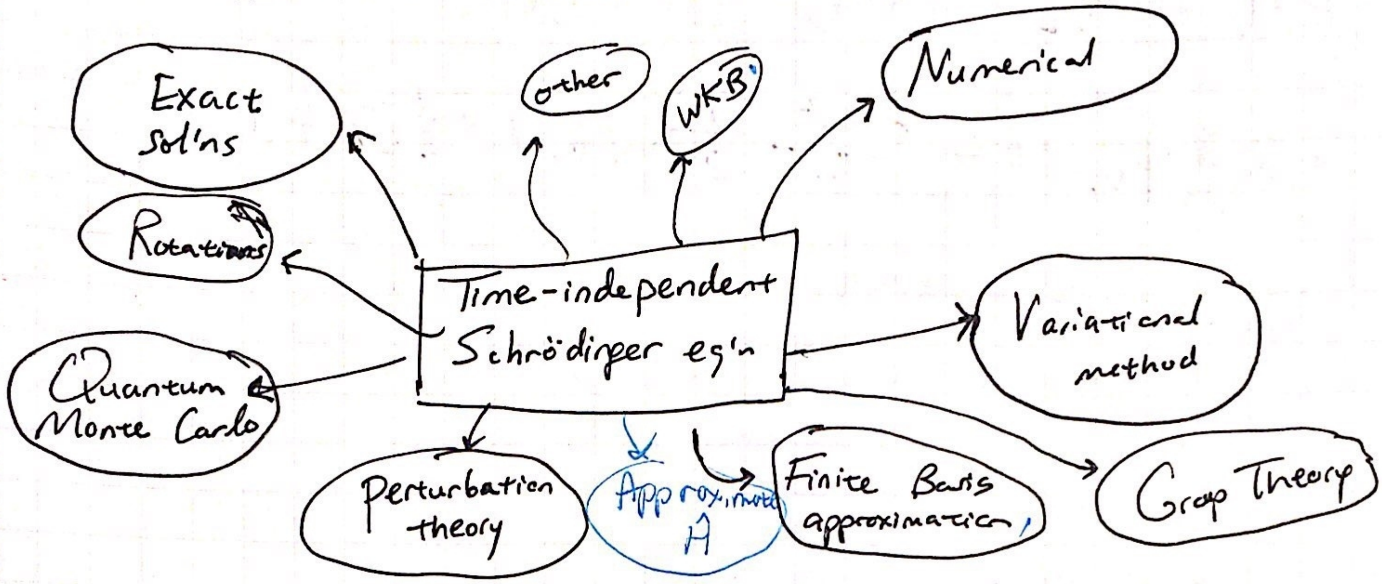
$$\hat{i} = -i\hbar A\frac{\partial}{\partial q}$$

$$i\hbar\frac{\partial}{\partial t}\psi(q,t) = \frac{1}{2}L\left(-i\hbar A\frac{\partial}{\partial q}\right)^2\psi(q,t) + E_q(q)\psi(q,t)$$

Use Ehrenfest's theorem: $\frac{d}{dt}\langle q\rangle = \langle \hat{i}\rangle$

$$\frac{d}{dt}\langle \hat{i}\rangle = \frac{-1}{L}\left\langle \frac{d}{dq}E_q\right\rangle$$

4 models for which there are exact sol'ns



WKB : Jefferies, Wenzel, Kramers, Brillow

Office hours 4 o'clock.