Pm Lecture: 13 10/4

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + \frac{1}{2} m\omega_0^2 x^2 \psi(x,t)$$

$$\Psi(x,t) = \left(\frac{m\omega_0}{\pi\hbar}\right)^{1/4} e^{i\frac{p(t)(x-\bar{X}(t))}{\hbar}} e^{-\frac{1}{2}\frac{m\omega_0}{\hbar}(x-\bar{X}(t))^2}$$

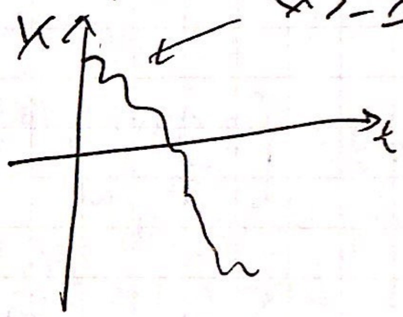
$$\frac{d}{dt}\bar{X}(t) = \frac{p(t)}{m}$$

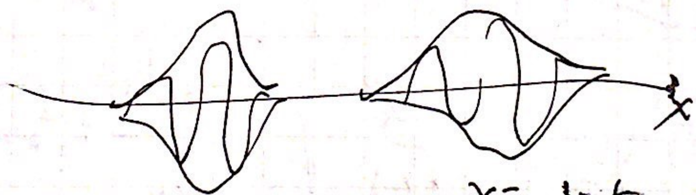
$$\frac{d}{dt}p(t) = m\omega_0^2 \bar{X}(t) \qquad \hbar\frac{d}{dt}\theta(t) = \frac{1}{2}\hbar\omega_0 - \frac{p^2(t)}{2m} + \frac{1}{2}m\omega_0^2\bar{X}^2(t)$$

$$\langle \hat{H} \rangle = \frac{1}{2}\hbar\omega_0 + \frac{p^2(t)}{2m} + \frac{1}{2}m\omega_0^2\bar{X}^2(t)$$

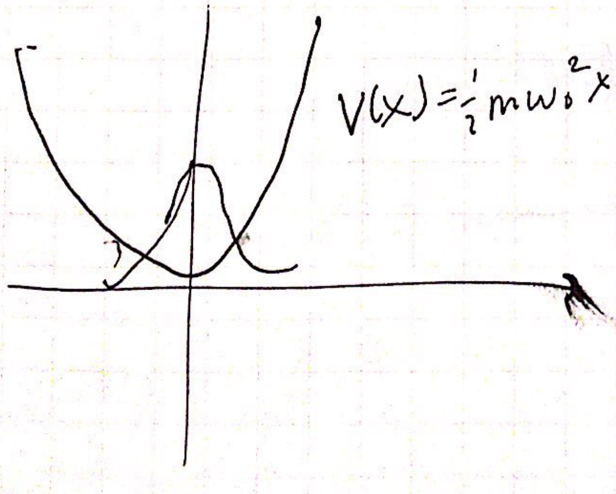
$$= \hbar\omega_0\left(\langle \hat{n} \rangle + \frac{1}{2}\right)$$

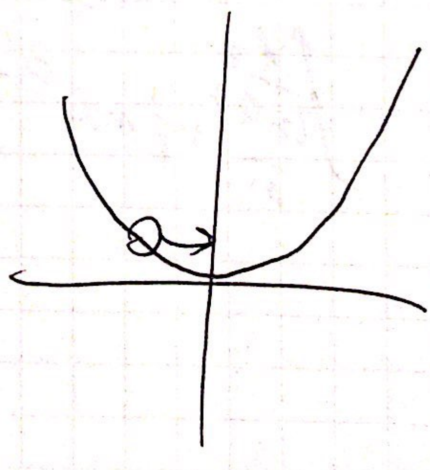
$$\langle \hat{n} \rangle = \frac{\frac{\hat{p}(t)}{2m} + \frac{1}{2}m\omega_0^2\bar{X}^2(t)}{\hbar\omega_0}$$





$V(x) = \frac{1}{2}m\omega_0^2 x^2$





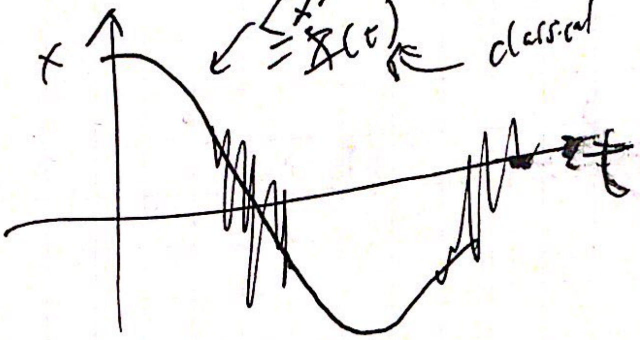
$$\frac{d}{dt}\langle x\rangle = \frac{\langle\hat{p}\rangle}{m}$$

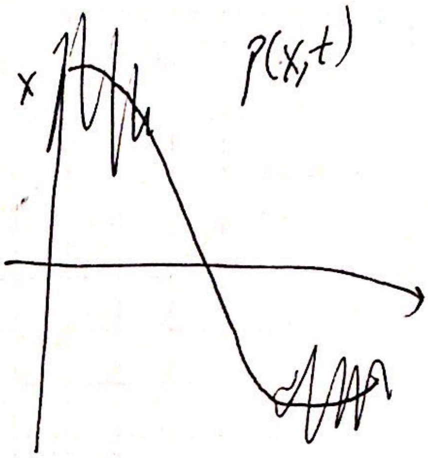
$$\frac{d}{dt}\langle\hat{p}\rangle = -m\omega_0^2\langle x\rangle$$

$$\frac{d}{dt}(\Delta x)^2 = \ldots$$

$$\frac{d}{dt}\langle (x-\langle x\rangle)(\hat{p}-\langle\hat{p}\rangle) + (\hat{p}-\langle\hat{p}\rangle)(x-\langle x\rangle)\rangle$$

$$\frac{d}{dt}(\Delta p)^2 =$$





LIGO Gravitational wave detection

$\Rightarrow$ use squeezing

$$\psi(x,t) = \left[\frac{2\alpha(t)}{\pi}\right]^{1/4} e^{-i\theta(t)}\, e^{i\frac{P(t)(x-\bar{X}(t))}{\hbar}}\, e^{-\alpha(t)[x-\bar{X}(t)]^2}$$

$$\frac{d}{dt}\bar{X}(t) = \frac{P(t)}{m}$$

$$\frac{(\Delta p)^2}{2m} + \frac{1}{2}m\omega_0^2(\Delta x)^2 = \ldots ?$$

$$i\hbar\frac{d}{dt}\alpha(t) = \frac{2\hbar\alpha^2(t)}{m} \ldots$$

$$\frac{d}{dt}P(t) = -m\omega_0^2\bar{X}(t)$$

$$\frac{d}{dt}\theta(t) = \left[\frac{\hbar^2\alpha(t)}{2m} + \frac{m\omega_0^2}{8\alpha(t)}\right] - \frac{P^2(t)}{2m} + \frac{1}{2}m\omega_0^2\bar{X}^2(t)$$

$$\int\frac{d\alpha}{\frac{2\hbar\alpha^2}{m} + \frac{1}{2}m\omega_0} = \frac{1}{i\hbar}dt$$

$$\alpha(t) = \frac{m\omega_0}{2\hbar}\left[\frac{\beta\cos(\omega_0 t) + i\sin(\omega_0 t)}{\cos(\omega_0 t) + i\beta\sin(\omega_0 t)}\right]$$

$\omega_0 t = 0$, $\alpha \to \frac{m\omega_0}{2\hbar}\beta$

$\omega_0 t = \frac{\pi}{2}$ $\alpha \to \frac{m\omega_0}{2\hbar}\frac{1}{\beta}$

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = \frac{-\hbar^2}{2m(t)}\frac{\partial^2}{\partial x^2}\psi(x,t) + \frac{1}{2}m(t)\omega_0^2(t)x^2\psi(x,t)$$

$$- F(t)\,x\,\psi(x,t)$$

$$\psi(x,t) = \left(\frac{2\alpha(t)}{\pi}\right)^{1/4} e^{-i\theta(t)}\, e^{i\frac{p(t)(x - X(t))}{\hbar}}\, e^{\alpha(t)(x - X(t))^2}$$

$$\frac{d}{dt}X(t) = \frac{p(t)}{m(t)} + (\quad)$$

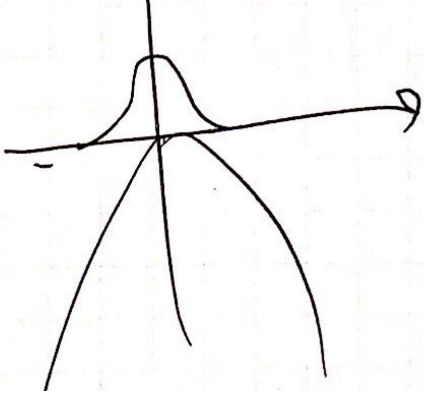
$$\hbar\frac{d}{dt}\theta = \left[\frac{\hbar\alpha(t)}{2m(t)} + \frac{m(t)\omega_0^2(t)}{8\alpha(t)}\right]$$

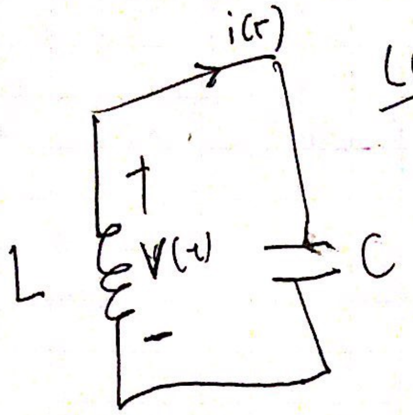
$$= \frac{p^2(t)}{2m(t)}$$

$$\frac{d}{dt}p(t) = -m(t)\omega_0^2(t)X + f(t)$$

$$i\hbar\frac{d}{dt}\alpha(t) = \frac{2\hbar\alpha^2(t)}{m(t)} - \frac{1}{2}m(t)\omega_0^2(t) + \frac{1}{2}m(t)\omega_0^2(t)X^2(t) - F(t)X(t)$$

$U(x) = \frac{1}{2}\omega_0^2 x^2$





LC circuit

$$i(t) = C \frac{d}{dt} V(t)$$

$$V(t) = -L \frac{d}{dt} i(t)$$

$$E = \frac{1}{2} L i(t) + C v^2(t)$$

$$C \frac{d^2}{dt^2} v(t) = \frac{d}{dt} i(t) = -\frac{1}{L} v(t)$$

$$\frac{d^2}{dt^2} v(t) = -\frac{1}{LC} v(t) \quad \omega_0^2 = \frac{1}{LC} \quad = -\omega_0^2 v(t)$$

$x(t)$ is like $Cv(t)$

$$\frac{d}{dt} x(t) = \frac{p(t)}{m} \qquad \frac{d}{dt} v(t) = -m\omega_0^2 (x)$$

$p(t)$ is like $i(t)$

$$\longrightarrow \frac{d}{dt} p(t) = \frac{1}{C} i(t) \qquad \rightarrow \frac{d}{dt} i(t) = -\frac{1}{L} v(t)$$

$$E = \frac{p^2(t)}{2m} + \frac{1}{2} m\omega_0^2 x^2(t) \longleftrightarrow E = \frac{1}{2} L i^2(t) + \frac{1}{2} C v^2(t)$$

$E = \hbar\omega$

$p = \hbar k$

$$E = \frac{p^2(t)}{2m} + \frac{1}{2} m\omega_0^2 x^2(t)$$

$$\hookrightarrow \hat{E}\psi = \frac{\hat{p}^2}{2m}\psi + \frac{1}{2} m\omega_0^2 x^2 \psi$$

$$i\hbar \frac{\partial \psi}{\partial t} = \frac{\left(-i\hbar \frac{\partial}{\partial x}\right)^2}{2m}\psi + \frac{1}{2} m\omega_0^2 x^2 \psi$$

$$E = \frac{1}{2} L i^2(t) + \frac{1}{2} C v^2(t)$$

$$\hat{E}\psi = \frac{1}{2} L \hat{i}^2 \psi + \frac{1}{2} C v^2 \psi$$

Scanned with CamScanner

$$\frac{i\hbar}{\partial t}\psi = \frac{1}{2}L(\quad \nabla^2\psi + \frac{1}{2}Cv^2\psi$$

$$\hat{P} = -i\hbar A\frac{\partial}{\partial v}$$

$$i\hbar\frac{\partial}{\partial t}\psi(v,t) = -\frac{1}{2}L\hbar^2A^2\frac{d^2}{\partial v^2}\psi(v,t) + \frac{1}{2}Cv^2\psi(v,t)$$

$$\psi(v,t) = \psi(v)e^{-iEt/\hbar}$$

$$E\psi(v) = -\frac{\hbar^2A^2}{2}\frac{d^2}{dv^2}\psi(v) + \frac{1}{2}(Cv^2\psi(v))$$

$$\psi(v) = e^{-\beta v^2/2}$$

$$\psi(v) = e^{-\beta v^2/2} = e^{-\frac{\frac{1}{2}Cv^2}{\hbar w_0}}$$

$$\beta = \frac{C}{\hbar w_0}$$

$$A = \frac{1}{\sqrt{LC}} = \omega_0^2$$

$$\hat{H} = \frac{-\hbar^2w_0^2}{2C} + \frac{1}{2}Cv^2$$

$$\hat{H} = \frac{1}{2}L\hat{i}^2 + \frac{1}{2}Cv^2$$

$$= -\frac{1}{2}\frac{(\hbar\omega_0)^2}{C}\frac{d^2}{dv^2} + \frac{1}{2}Cv^2$$

quantum mechanical model for LC circuit.

quantized LC circuit